


A review of the effects of cyclic contact loading on fretting fatigue behavior

Advances in Mechanical Engineering
2020, Vol. 12(9) 1–17
© The Author(s) 2020
DOI: 10.1177/1687814020957175
journals.sagepub.com/home/ade


F Abbasi¹ , GH Majzoubi² and J Mendiguren¹

Abstract

A damage phenomenon called fretting fatigue frequently takes place when two contact bodies are clamped together under a normal contact load along with a small-scale oscillatory motion due to cyclic loading. In contrast to the constant contact loading, less attention has been paid to variable contact loading which was technically reviewed in this study. Emphasis was placed on the efforts made over the past decade and the future challenges including nonlinear effects of contact loads, friction, frequency, slip amplitude, wear, and contact mechanic are discussed extensively. It was revealed a need for new fatigue and contact mechanics models by identifying the aforementioned missing parameters.

Keywords

Fretting fatigue, variable contact loading, contact mechanics, frequency, wear

Date received: 8 June 2020; accepted: 17 August 2020

Handling Editor: James Baldwin

Introduction

Fatigue, fracture, wear, corrosion, creep, and fretting are among the foremost common mechanisms of failure. These failure modes are usually taken under consideration within the design process of any engineering component. Fretting fatigue is a combined action of fretting and fatigue failures, which occurs when two contacting surfaces are subjected to a normal stress and an axial cyclic loadings at the same time. Fretting fatigue occurs due to the small amplitude sliding motion between the two mutually clamped surfaces. Fretting fatigue is a destructive failure which can reduce the life of a component by a factor of typically between 2 and 3 although, factors as high as 10 have also been reported.¹ Fretting fatigue occurs widely in various mechanical components, such as in dovetail of turbine blades, bolted and riveted joints, bearings, cables, etc.²

Fretting fatigue has been the subject of numerous investigations over the past few decades, various techniques have been proposed to enhance fretting fatigue life, different methods have been suggested to estimate fretting fatigue life and most of aspects of fretting fatigue mechanism such as crack initiation location, crack

propagation direction and fretting fatigue life have been explored.^{3–19} However, majority of the previous works have been performed under a constant normal load and fewer investigation have been accomplished on fretting fatigue under variable contact loading. This is while most of engineering applications such as dovetails of turbine blades experience cyclic contact loading. The blade/disk interfaces of gas turbine engine was identified as the critical location for fretting fatigue damage by Gowda.²⁰ During rotation of engine, this interface is subjected to fretting induced by the aero-dynamical high frequency vibrations performing on the blade and the centrifugal load as shown in Figure 1. Engine vibrations along with a high unsteady aerodynamic loads

¹Department of Mechanical and Industrial Production, Mondragon Unibertsitatea, Loramendi, Mondragon, Spain

²Mechanical Engineering Department, Faculty of Engineering, Bu-Ali Sina University, Hamadan, Iran

Corresponding author:

F Abbasi, Advanced Material Processes Research Group, Department of Mechanical and Industrial Production, Mondragon University, Loramendi 4, Mondragon, Gipuzkoa 20500, Spain.
Email: fabbasi@mondragon.edu



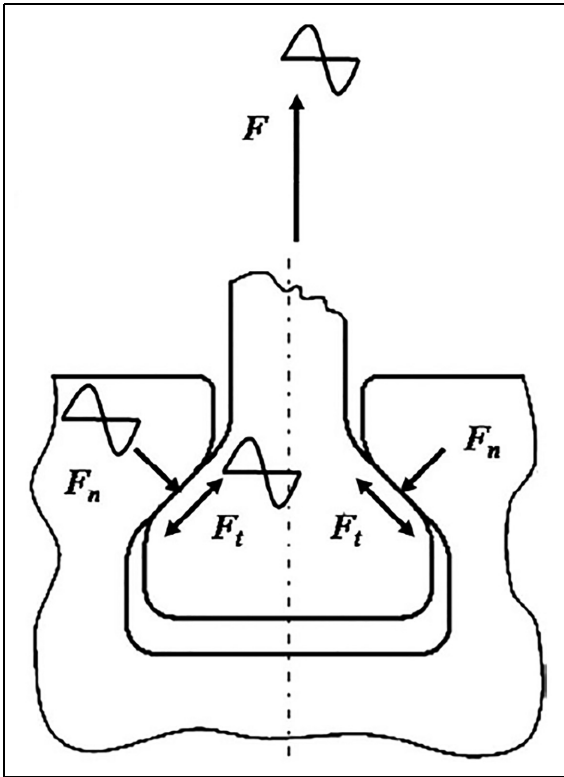


Figure 1. The blade/disk contact and loading condition of a turbine engine.

can result in a high loading frequency (>500 Hz) at the contact interface.²¹

The frictional force creating the relative oscillatory motion can be less than that required to create gross sliding over the contact area as a whole. This results in a small central zone of no relative motion within the two contacting surfaces known as the stick zone. This type of contact is known as partial slip contact. The relative motion within the slip zone of the contact gives rise to wear that in turn brings about crack initiation at the border of slip and stick zones within the contact region. Vingsbo and Soderberg²² produced a series of fretting fatigue maps to identify the three contact regimes (Stick, stick-slip, and gross slip) in terms of the two variables, normal load and displacement amplitude, or normal force and shear load as shown in Figure 2(a). From the experimental results reported in the literature, Vingsbo and Soderberg²² estimated the transition displacement amplitudes between the three regimes. For instance, the transition between the partial slip regime and gross slip could occur at the displacement amplitudes between 3 and 25 μm . However, this range of amplitudes is not a rigid range for maximum fretting fatigue damage, that is, it generally depends on contact type, specimen geometry and material, loads. Using the maps of fretting fatigue makes it possible to

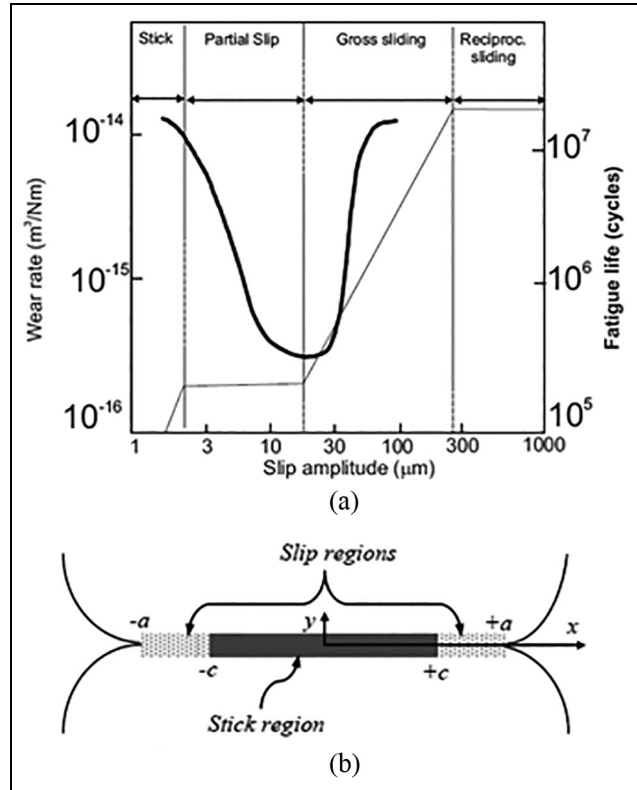


Figure 2. Schematic view of the fretting map.^{22,23}

identify the value of slip amplitude at which the minimum fretting life occurs.

As Figure 2(a) shows, that minimum fretting fatigue lives occur in the partial slip regime. In this regime, sharp peaks in shear stress occur at the stick-slip boundary as shown in Figure 2(b). In the regime a central stick region is surrounded by an outer slip region. Wear occurs in the slip region but little surface damage occurs in the center of the contact. Cracks initiate from the worn damage region due in part to micro changes in surface geometry and the stress concentration resulting from such changes. The most damaging contact regime in fretting fatigue is well known to be the partial slip regime where the fretting fatigue life decreases with increasing slip amplitude.²⁴ The reduction in life with increasing slip amplitude in partial slip regime can be explained by the shear traction distribution on the contact interface. The total shear force Q applied to the contact interface is given by the following equation²⁴:

$$Q = b \int_{-a}^a \tau(x) dx \quad (1)$$

Where $\tau(x)$ is the shear traction at point x , b is the specimen width, and a is the contact semi-width. Until gross slip is prevailed, increasing in slip amplitude

results in increasing frictional force, so the trailing edge of the contact stresses scales up, thus reducing the predicted fatigue life. If slip amplitude is increased so that the gross slip condition is exceeded ($Q > \mu P$), the frictional force cannot increase any more so that the trailing-edge stresses and predicted fatigue life are largely unaffected by further increases in slip amplitude. At this point, the predicted fatigue life has reached a minimum.²⁴

The increase in fretting fatigue life in the gross sliding regime can be explained by “crack wipe-out mechanism”.²⁵ According to this mechanism, in the gross slip regime the micro-cracks are eroded before they get the chance to propagate. Fretting map developed by Vingsbo and Soderberg²² provides a clear picture of the relative damage of the three contact regimes. Nevertheless, this map is comparatively old and its accuracy is under question.²⁶ As an example, transition to reciprocating sliding wear for the low wear rate regime at low displacement amplitudes has been largely debunked by recent studies.^{26,27} At low amplitudes of displacement, a “stick regime” is found with no crack initiation which is somewhat disagrees with the Cattaneo-Mindlin-Ciavarella-Jaeger²⁸ classical map for which at the edges of the contact the slip should immediately initiate for arbitrary small tangential forces.

The eventual objective of fretting fatigue investigations is to predict unexpected in-service failures and safeguard the systems against them. To this end, significant efforts in the literature has been placed on fretting fatigue life estimation using experimental and numerical methods. Both fracture and damage mechanics have been utilized in the literature and the results usually are validated by experiment. The wear parameters the most well-known of which is the Ruiz parameter²⁹ have specifically been developed for fretting fatigue. On occasions, cumulative damage laws such as Miner³⁰ have also been used to predict the behavior of fretting fatigue under constant contact loading. There is still, however, no widely accepted technique for predicting fretting fatigue and current researches are still at the assessing stage of these tools. Nevertheless, the effect of variable contact loading has largely been ignored in the investigations. Most of the research works in this field has up to now been performed by Mall et al.³¹ and Abbasi and Majzoubi.^{32–39} Contributions have also been made by Hojjati-Talemi et al.²³, Xin et al.,⁴⁰ Ciavarella et al.⁴¹ and some others.^{42,43}

The major objective of the present article is to review the most recent efforts in the literature handling fretting fatigue under variable contact loading. The effects of the main fretting variables, which are directly associated with the features of the testing apparatus, are described. The state of the art knowledge in contact mechanics regarding the variable contact loading is also

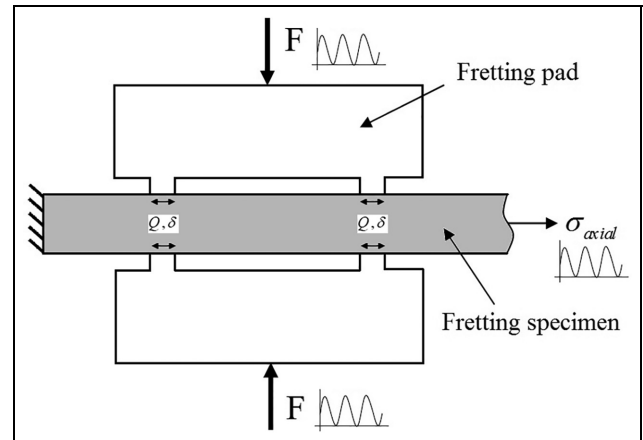


Figure 3. Schematic of a typical fretting fatigue configuration.

briefly presented. Finally, the recent progresses in the field of fretting fatigue is outlined, and recommendations for future are suggested.

Testing apparatus

The early fretting fatigue researches were conducted using various test apparatuses. However, fretting fatigue testing has become increasingly standardized. Two types of loads are present within a fretting fatigue setup, global and local loads. Global loads include the normal and shear loads, which are applied to the pad, and the bulk axial load, which is applied to the sample as shown in Figure 3. The shear load is usually dependent on the amplitude of bulk axial load. The local loads such as shear stresses distributed over the contact interface are the main source to crack initiation.

The geometry of fretting contact has significant influence on the state of local stresses. It is highly desirable to know the stress distribution at the contact interface as this will facilitate the estimation of fretting fatigue life. This ends up in the utilization of idealized contact geometries during testing where, Hertzian, for example, spherical point or cylindrical line contacts are the most frequently used contact geometries. Analytical solution for the stress state within the contact zone is available for the commonly observed fretting loads. The rounded punch contact is also being increasingly employed in applications such as dovetail joints for which significant research efforts have been devoted in recent years. In 1960's cylindrical fretting pads were increasingly used the stress distribution within the fretting contact zone were obtained by analytical solutions. The fretting global loads could be measured and controlled and hence fretting fatigue testing has moved increasingly toward standardisation.⁴⁴

In modern fretting fatigue testing, it is necessary to obtain as much control over the conditions in the

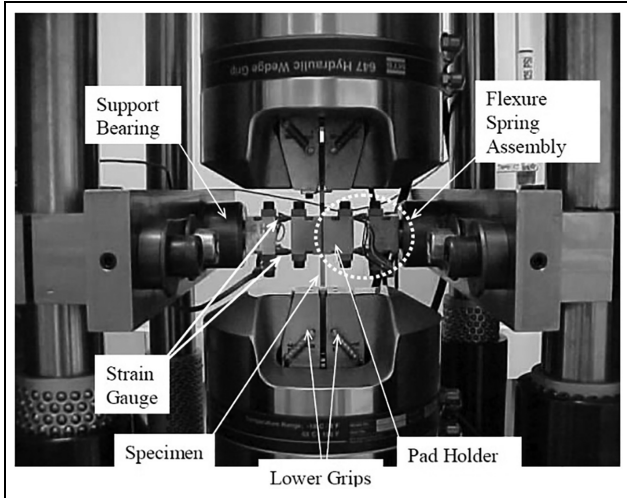


Figure 4. AFIT Multi-axis servo-hydraulic test machine setup for cyclic contact loading.⁴⁷

contact region as possible. Finite element analysis (FEA) has been used to design a series of experiments having a specific stress distribution generated via the application of global loads.⁴⁵ The bulk axial load is usually supplied and controlled by a single actuator which is typically the main actuator of the hydraulic testing machine. The normal load is supplied by some sort of spring arrangements such as bevel washers, spring, bolt-nut connections or proving ring. The load is measured via strain gauges bonded to the proving ring or a load cell. We do not intend to present a detailed review of fretting fatigue apparatus and testing practices in this paper. However, the testing apparatus which have been designed and developed for fretting fatigue testing under variable loading condition are briefly addressed in this section.

A coupon scale test setup developed at U.S. Air Force Institute of Technology (AFIT) is illustrated in Figure 4. The test machine consisted of a rigid steel box frame, two 5 kN side horizontally opposed actuators and a 50 kN lower vertical servo-hydraulic actuator. All actuators were controlled using Instron 8800 function generator software.⁴⁶ The maximum contact load frequency created by this setup was about 40 Hz, due to the limitation of the hydraulic actuators. Nonetheless, in most of practical applications such as aero engines, which are subjected to fretting fatigue, the vibration frequency is much greater and may exceed even 200 Hz.³²

A full-scale biaxial rig illustrated in Figure 5⁴⁸ was developed at Oxford University. The test-rig can simulate the effects of blade vibration, disk expansion force, and centrifugal loading. Figure 5 shows a pair of blade specimens which have been tested using this apparatus. In actual dovetail attachment, the contact zone is rather flat with rounded corners; the variation of the forces

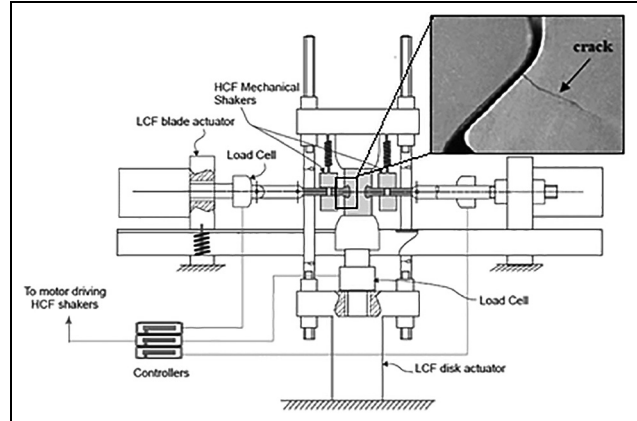


Figure 5. Schematic view of a biaxial fretting fatigue testing machine.⁴⁴

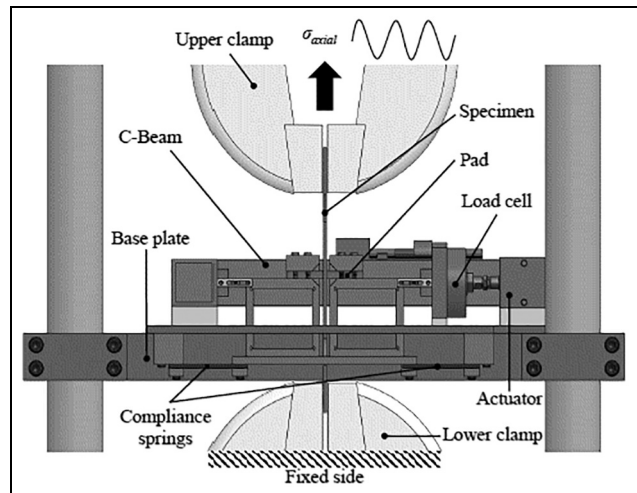


Figure 6. Schematic view of the fretting fatigue with adjustable compliant springs mounted on universal servo-hydraulic machine.⁴⁹

induces bending moments and the contact area changes in a very complex manner. The initiation of fretting fatigue induced cracks at the edge of the contact may clearly be seen in Figure 5. The main limiting issue with this setup is that it is a full-scale test rig and therefore, the tests are expensive and time consuming and the results are complex to be interpreted.

De Pauw et al.⁴⁹ designed and equipped a servo-hydraulic load frame with a fixture as depicted in Figure 6. A 100 kN hydraulic actuator is used to apply the axial fatigue load in the dog-bone specimen and the normal contact load is applied by a single servo-hydraulic actuator as shown in the figure. The tangential force, between the pads and specimen is generated by means of leaf springs. The tangential load, which is proportional to the fatigue load comes from the elastic deformation of the dog-bone specimen and compliance

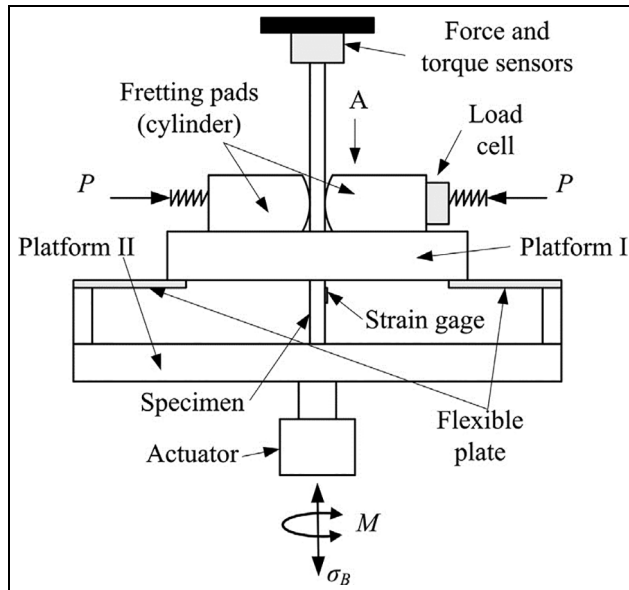


Figure 7. A schematic view of the biaxial fretting fatigue testing machine.⁴⁰

of the leaf springs. A lateral load cell is attached to the C-beam in order to measure the contact load directly. The induced tangential load is measured by a strain gage attached to the compliance springs. The main limitation of this test rig is that it uses only one actuator to apply contact load. This may cause asymmetry of contact stresses at the two contact interfaces. Furthermore, the maximum contact load frequency obtainable by the device is 10 Hz, which varies in-phase with the bulk axial fatigue load.

An electro-hydraulic servo tension-torsion fatigue test system was modified by Xin et al.⁴⁰ to simulate the oscillatory contact loading as shown in Figure 7. As the figure indicates, in addition to the bulk stress σ_B , the specimen is also subjected to a torsional moment, M . The moment acting on the contact interface between the specimen and pads alters the distribution of the normal contact pressure. The loading arrangement, gives rise to cyclic normal load to be generated in the device. By changing the effective gauge length (contact point of the fretting pads on the specimen) or changing the thickness of the flexible plate, the slip amplitude can be adjusted. The loading condition generated by the setup is closer to the working condition of the real engineering cases as stated by the authors.⁴⁰

More recently, a simple coupon scale testing apparatus shown in Figure 8(a) has been designed and developed by Abbasi et al.³² to simulate the fretting behaviour of materials under cyclic contact loads. The main feature of the test rig is that a simple synchronized electro-mechanical system (a motor driven mechanism) is employed for generating the cyclic

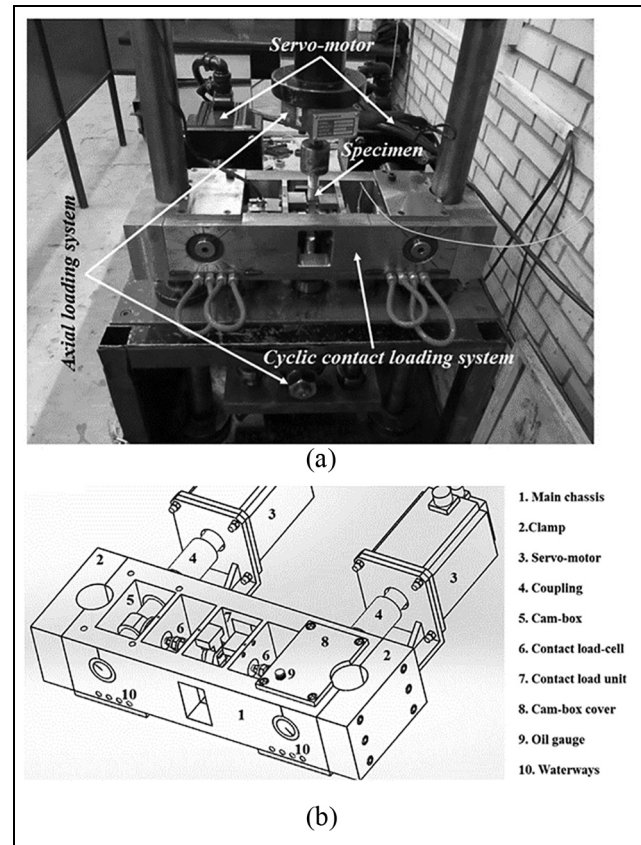


Figure 8. The test rig developed by Abbasi et al.,³² (a) general view and (b) schematic view of the device.

contact loads, rather than using hydraulic actuators which are complex, expensive and low frequency as shown in Figure 8(b). The capabilities to adjust the normal load independent of the axial loading and to change the phase between normal and far field axial loads are other useful features of the device. The device is additionally capable of generating cyclic loading with frequency of 0 to 200 Hz and amplitude of 0 to 10 kN. This range of frequency is the case for many engineering applications exposed to fretting fatigue such as dovetail joints in aero engines. A detailed description of the test-rig can be found in.³²

Summary of previous works

The analysis of fretting fatigue requires a thorough knowledge of multiaxial loading, contact mechanics, tribology, metallurgy, fatigue, and fracture, etc. These subjects involve a number of parameters such as bulk loads, local contact loads, contact geometry, frequency of applied loads, material behavior, slip amplitude, friction, and surface treatments that can affect fretting fatigue behavior of materials. These variables are not independent of each other. For example, the formation

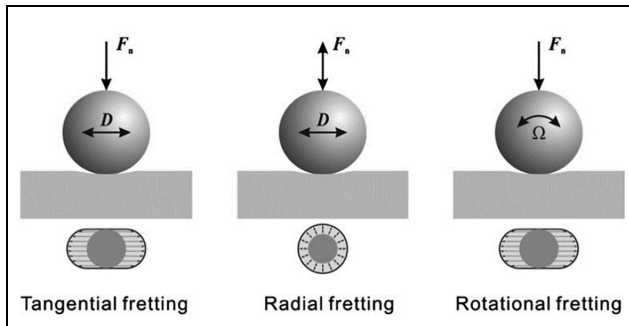


Figure 9. Schematic diagram of fretting modes.

of fretting debris will affect the localized friction behavior of the contact. Despite this dependency, the variables can be divided into two groups, global and local variables. Other factors that may play a role in fretting fatigue crack initiation are persistent slip bands, fretting damage, localized heating effects at asperities and also the presence of debris.^{9,50}

The application of contact loads both in a global and local context is what distinguishes fretting fatigue from plain fatigue. The application of the contact loads gives rise to localized stress concentrations at the edge of the contact surfaces between the pad and the specimen known as trailing edges. Although, the type, magnitude and distribution of the local stresses are influenced by the shape of the contacting components they are still severely dependent on the applied contact loads, that is, normal and shear force. Therefore, it is often difficult to characterize the effect of each variable individually on fretting fatigue life, as most of the fretting fatigue variables are interrelated. For such multi-variables investigations sophisticated test apparatus is required. In spite of this, attempts have been made to explore the relative importance of individual fretting fatigue variables on fretting fatigue life of component's.⁴⁵ In the next sections, the major investigations on cyclic loading on the fretting fatigue behaviour of material reported in the literature are reviewed.

Katholieke Universiteit Leuven

The displacement field of an arbitrary vibration can be divided into three basic fretting modes characterized by a tangential, radial and rotational fretting as shown in Figure 9 for a ball-on-flat contact condition. Laboratory fretting experiments reported in literature up to now have focused on this mode I.^{22,51} Under fretting Mode II, the contact area oscillates between a banded limit under an oscillating normal load. This type of fretting takes place in practical applications such as surgical implants (e.g. dental restorations), aircraft components and automobile engine components.^{44,52,53} Finally, fretting mode III occurs under a

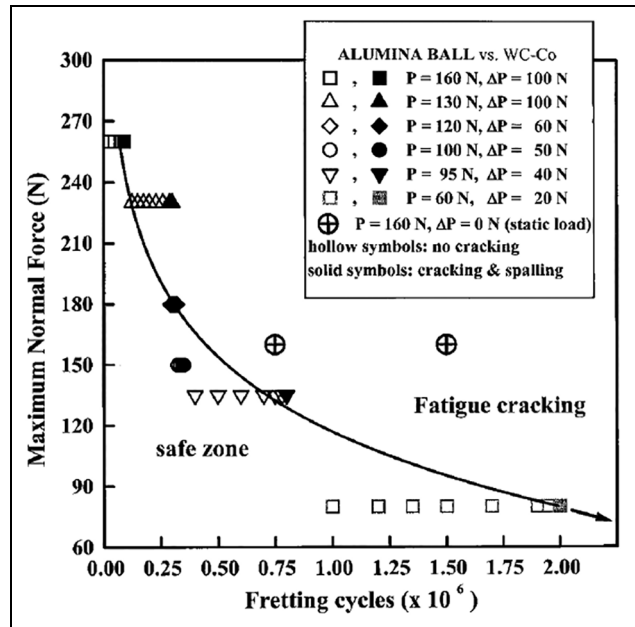


Figure 10. Fretting map for alumina balls under fretting Mode II.⁴³

twisting relative displacement so that slip occurs in a contact zone at the contact periphery whereas the inner zone remains locked. Huq et al.^{42,43} examined the effect of cyclic contact load on degradation mechanism in an alumina versus a WC-Co ball-on-flat contact under fretting mode II. Before the beginning of the test, a static normal load of 150 N was applied to the contact. During the tests, that normal load was subjected to a superimposed sinusoidal oscillation with a frequency of 35 Hz at a peak-to-peak amplitude of 200 N.

The results indicated the existence of spalling and cracking in the slip area on the alumina sample. Analysis of the wear scar using EDS revealed the presence of oxygen in the slip area, whereas the presence of oxygen was not found in the central region. They concluded that oxidation in the slip region takes place as a result of the periodic exposure of the corresponding contact region to air. It was found that, the central locked region of the contact interface does not oxidize during fretting mode II since the central part of the wear track remains permanently in contact with the counter body.

Huq et al.^{42,43} conducted several tests for different numbers of fretting cycles at different normal loads and load amplitudes in order to determine the number of fretting cycles required to form a threshold crack or spalling dimension of 20 μm , which is of the order of several grain diameters. Their results led to a fretting map shown in Figure 10, where variation of maximum normal force versus the number of fretting cycles is plotted. The solid markers in Figure 10 indicate the

tests carried out at a specific normal force where, for the corresponding number of fretting cycles, a threshold cracking of $20\ \mu\text{m}$ was found on the surface of the alumina ball. This fretting map can be a useful tool for design purposes. For making a comparison between the constant and cyclic loadings, two fretting tests were conducted under the static normal loading of 160 N. The normal load was supplied by an alumina ball loaded on a WC-Co flat contact. The fretting tests were performed under Mode II for durations of $0.75e6$ and $1.5e6$ cycles. Cracking was not detected under static conditions either on the surface of the alumina balls or on the WC-Co plate. These results suggest that some precautions should be taken into account in designing components for contact conditions exposed to oscillating contact loads.

Air Force Institute of Technology

In recent years, much attention has been paid to fretting fatigue of aerospace components in order to prevent catastrophic air disasters particularly heavy casualties in air crashes. The estimates by the U.S. Air Force indicate that one in six of all in-service high cycle fatigue related engine failures were due to fretting fatigue and galling. An estimated \$ 20 Million spends annually on preventive measures by the US Air Fore.⁴⁴

At Air Force Institute of Technology (AFIT) many studies have been conducted on fretting fatigue behavior of material under cyclic contact loading condition. Iyer and Mall⁵⁴ studied the relation between contact load and frequency of the bulk axial load for Ti-6Al-4V. They reported that contact pressure reduces the fretting fatigue life for the axial load frequency of 1 Hz. However, no apparent effect contact load on fretting fatigue life was found for the cyclic axial load frequency of 200 Hz. In all tests, the trailing edge of the contact interface was found as that crack nucleation location. In another study by Lykins et al.⁵⁵ they observed that fretting fatigue life was independent of axial load frequencies of 1 Hz and 200 Hz. They concluded that, the major effect of axial load frequency on fretting fatigue is its effect on the level of oxidation that is allowed to occur in the contact region, as the oxidation process is time dependent. Heating effects at high frequencies may also be seen due to friction.

Jutte⁴⁷ examined the effect of cyclic contact loading on fretting fatigue behaviour of titanium alloy. It was reported that the cyclic normal contact loading has more damaging effect on fretting fatigue life than the constant normal contact loading. As Figure 11 shows, fretting fatigue life is lower for the tests with fluctuating contact loading than that for the tests with corresponding higher and lower constant contact loads. Figure 11 shows that fretting fatigue lifetime of specimens under variable contact loading is on average 70% lower than

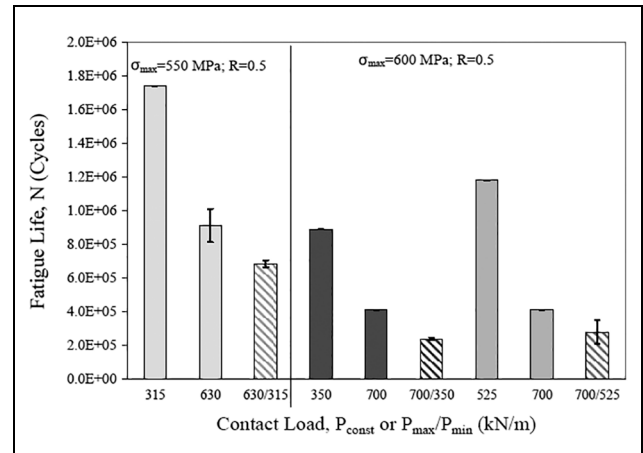


Figure 11. Fretting fatigue life for various contact loading conditions.⁴⁷

that for the corresponding lower constant contact loading and 33% lower than that for corresponding higher constant contact loading.

Lee⁵⁶ explored the effect of shot-peening on fretting fatigue response under both constant and cyclic contact loads for Ti-6Al-4V alloy. The experiments were done under four contact load frequencies of 0, 2.5, 10, and 30 Hz. The frequency of applied axial loads was 10 Hz. In all tests, fretting fatigue cracks were found to initiate near the trailing edge of the contact. Cracks initiated from the surface of un-peened specimens and from the interior points of shot-peened specimens. A modified shear stress range (MSSR) was examined to seek out their possible role in predicting fretting fatigue life. No correlation found between fretting fatigue mechanisms and contact load conditions, and shot-peening improved fretting fatigue life despite the contact load conditions. Lee⁵⁶ also showed that for both the constant and cyclic contact loadings, the MSSR parameter was effective in predicting the fretting fatigue life and crack nucleation location and its orientation.

The effect of phase difference between tangential and bulk axial loads on fretting fatigue response of Ti-6Al-4V was studied by Lee and Mall.⁵⁷ It was found that although both the tangential load and the relative slip varied with phase difference, however the effect of phase difference on the fretting fatigue life was negligible.

The effect of phase difference between contact and axial loads on fretting fatigue behavior of Ti-6Al-4V alloy was studied by Almajali.⁵⁸ The frequency was fixed at 10 Hz for both the contact and axial loads. The result reported by Almajali⁵⁸ showed that the fretting fatigue life for in-phase cyclic contact loading was the same as that for the constant contact loading. However, for the out-of-phase loading fretting fatigue life was increased, with respect to in-phase condition, from 20%

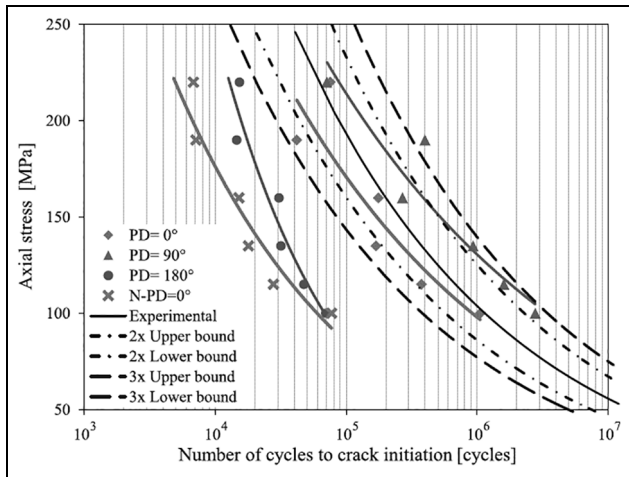


Figure 12. Fretting fatigue crack initiation lifetime for variable contact loading and different phase differences between axial and tangential loads (PD: Phase Difference).²³

to 30% in the low cycle regime and up to 150% in the high cycle regime. Al-Noaimi⁵⁹ studied the effect of phase angle between axial and normal contact loads on fretting fatigue behavior of Ti-6Al-4V alloy. He found that for the same applied axial load, fretting fatigue lifetime increased with phase angle. It was also reported that under variable contact loading conditions, the induced tangential load had the same frequency and remained in-phase with the applied axial load.

Ghent University

In fretting fatigue contact zones, the state of stress at and near the contact interface is extremely influenced by combination of axial, tangential and contact loads. The multiaxial nature of stress at contact interface can be severely influenced by phase difference between axial, tangential, and contact load. Using an uncoupled damage evolution model, Hojjati-Talemi et al.²³ studied the effect of various phase difference between axial and tangential loads on fretting fatigue behavior of aluminum 2024-T3 alloy. They also investigated the effect of variable contact loading on fretting fatigue response and crack initiation lifetime of aluminum 2024-T3 alloy. Figure 12 depicts a comparison between the predicted fretting fatigue crack initiation lifetimes for various phase differences and the experimental results. It is seen that by increasing the phase difference from 0 to 90 degrees the crack initiation lifetime improves. For PD = 180°, however, the crack initiation lifetime was predicted to be less than that for the 0 degree phase difference. Furthermore, as depicted in Figure 12, for variable contact loading, it was found that the initiation lifetime decreased almost at the same rate as it did for PD = 180°.

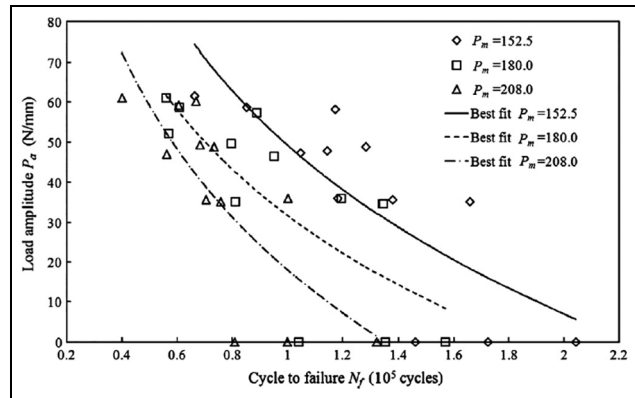


Figure 13. Variation of load amplitude, P_a , versus fatigue life, N_f for different mean load, P_m .⁴⁰

Beijing Institute of Technology

A methodology was proposed to examine the fretting fatigue mechanism of AA6061-T6 aluminum alloy under cyclic loading condition by Xin et al.⁴⁰ From their study, it was found that the variable normal contact load will enlarge the slip area of the contact interfaces, making the crack initiation location contracts toward the center of the contact area. Variation of load amplitude, P_a , versus fatigue life, N_f , for different mean load, P_m , as reported by Xin et al.⁴⁰ is shown in Figure 13. As it is seen from the figure, for the same mean contact load level, fretting fatigue life declines as the contact load amplitude increases. Similar trend is observed for the mean contact load, that is, for the same contact load amplitude, fretting fatigue life diminishes as the mean contact load increases. From their study, Xin et al.⁴⁰ showed that the variable contact load will intensify the effect of the fretting on fatigue life, making the fretting fatigue life less than those in constant contact loading conditions.

Bu-Ali Sina University

Recently, Majzoobi and Abbasi^{32–39} attempted to identify the relative importance of several variables on fretting fatigue life of Aluminum alloy 7075-T6 under cyclic contact loading. The variables were contact load frequency,³³ phase difference between axial and contact loads,³⁴ peak pressure,³⁵ contact geometry,³⁶ shot-peening³⁷ and temperature.³⁸ The major findings by Majzoobi and Abbasi in their investigation on fretting fatigue response under cyclic contact loading are briefly addressed in the following sections.

Effect of contact load frequency. Fretting fatigue response of Al7075-T6 at different contact load frequencies was studied using an innovative test-rig shown in Figure 8. The results showed that fretting fatigue lifetime for the

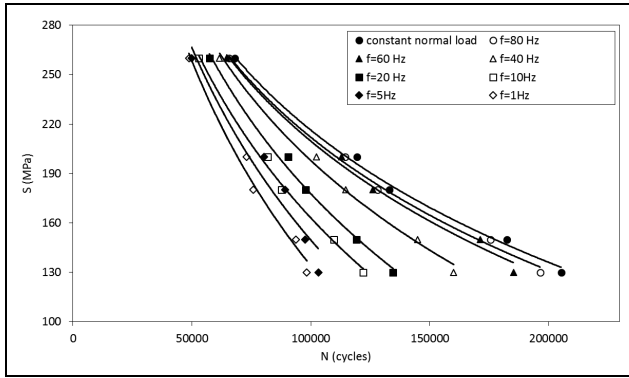


Figure 14. S-N curves for different contact load frequencies.

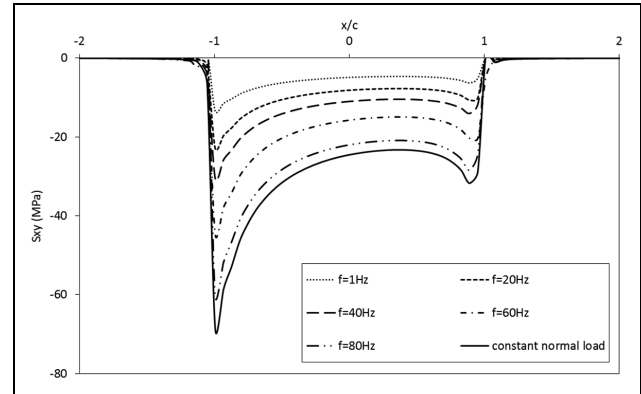


Figure 16. Distribution of shear stress along the contact interface for different contact load frequencies.

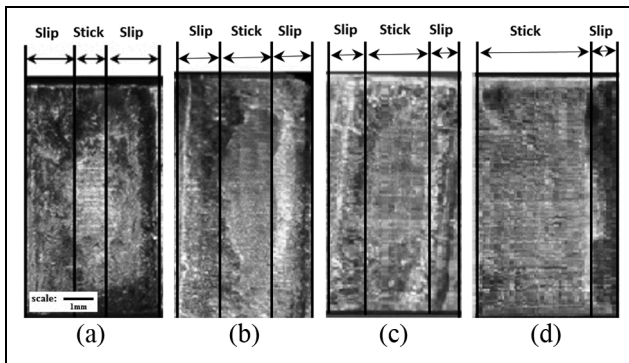


Figure 15. Optical microscopy images of slip area for different contact load frequencies: (a) $f = 1$ Hz, (b) $f = 20$ Hz, (c) $f = 80$ Hz, (d) constant contact load.

case of cyclic contact loading was less than the corresponding life under constant contact loading at the same axial and contact load levels as shown in Figure 14. As it is seen from the figure, fretting fatigue life sharply improves particularly for low stress levels, with increasing the contact load frequency. From Figure 8 it is also clearly seen that the fretting fatigue life at $f = 80$ Hz converges to its corresponding life at constant contact load condition.³³

The contact load frequency is a determining parameter in fretting fatigue loading. The parameter significantly affects the size of the stick and slip regions, as does the magnitude of shear load. Optical microscopy observation of the fretting scar clearly showed that partial slip regime takes place at the contact interface as shown in Figure 15. As it is seen from the figure, under cyclic contact loading condition, the slip region size is larger and its extent is decreased with the increase in contact load frequency. In particular, the size of slip zone at the frequency of 1 Hz is about 1.67 mm (Figure 15(a)) which is substantially greater than the slip zone size at the frequency of 80 Hz which is only 1.12 mm (Figure 15(c)). Larger slip region indicates that larger

relative displacement occurs under cyclic contact loading condition especially for lower contact load frequencies. This is particularly the case for lower contact load frequencies as oxidation is a time dependent process. It is understood that the size of contact region undergoes oscillation due to the fluctuation of the normal contact load during the process of loading, making the extent of the contact zone to be unstable.

The major drawback of loading condition in Majzooobi and Abbasi³² test apparatus is that the shear force are dependent on the stiffness of the bridge type fretting pad and also axial strain produced in the dog bone specimen. Thus, the level of shear force varies accordingly with oscillating the contact load in-phase with the axial load. Regardless of the stress time-history, under high contact load frequencies, the maximum friction force amplitude produced at the contact interface will be nearly the same as that for constant contact loading condition. On the contrary, if the contact load frequency is low, there will be no net friction force at the contact interface since the tangential friction force is released alternatively when the contact load decreases to its lowest level. This was confirmed by the results of numerical simulation shown in Figures 16 and 17 in which the slip region size declines and the shear stress generated at the contact interface increases with the increase in the contact load frequency.³³ This may also be a possible explanation for the smaller slip area at higher contact load frequencies in Figure 15.

Majzooobi and Abbasi³² used the fretting map of Vingsbo and Soderberg,²² shown in Figure 2, to explain the results of their fretting fatigue tests under cyclic contact loading. Vingsbo and Soderberg²² demonstrated that small change in slip amplitude and subsequently wear can lead to a large change in life. The increase in slip amplitude experienced during the application of cyclic contact loading will lead to increase in wear and thus possibly to life reduction.

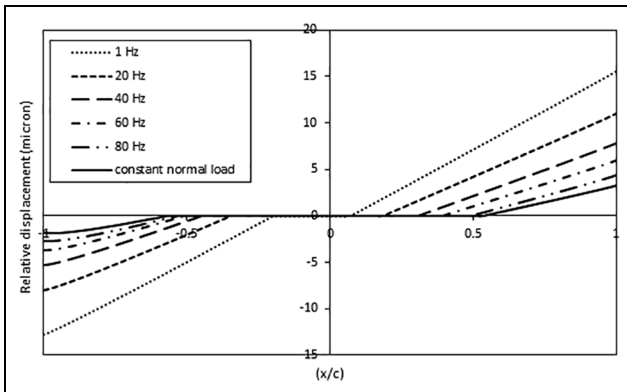


Figure 17. Distribution of relative displacement along the contact interface for different contact load frequencies.

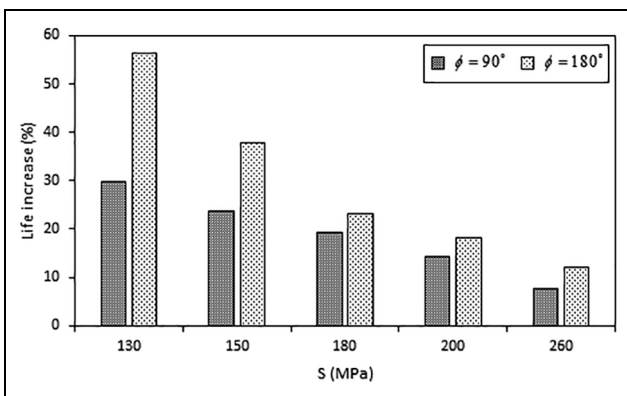


Figure 18. Percentage of fretting fatigue life improvement versus working stress for various phase differences, with respect to in-phase loading.

Effect of out of phase loading. Abbasi and Majzooobi³⁴ compared the results of the constant contact loading tests, with those of in phase, 90° and 180° degrees out-of-phase loads for the same frequency. The results are shown in Figure 18. It is seen from the figure that fretting fatigue lifetime has improved, with respect to the in phase loading, by 8% to 30% and 12% to 56%, depending on working stress, for phase differences of 90° and 180°, respectively. They concluded that the domination of gross slip regime at the contact interface along with the lower contact stresses are the main reasons for improvement of fretting fatigue life under out-of-phase loading condition.

The angle of crack growth is can provide useful information about one fretting fatigue mechanism. Fretting fatigue cracks initiate by shear stresses usually at an angle to the contact surface. Crack initiation is usually dominant by Mode II of fracture. The crack will then gradually changes direction until it grows sufficiently that the contact stresses no longer influence its

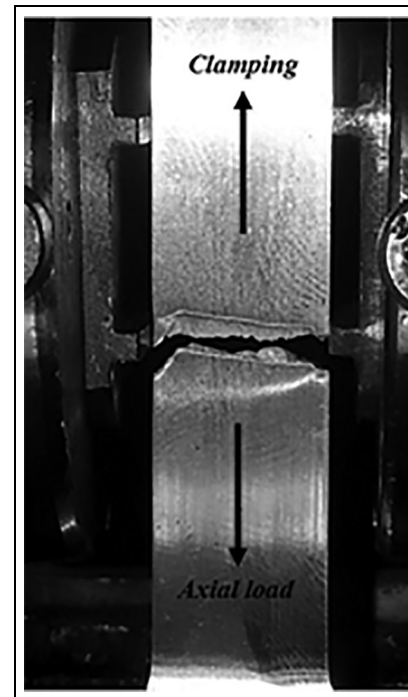


Figure 19. Typical fretting fatigue crack trajectory in Abbasi and Majzooobi experiments.

growth. At this point, the crack will grow perpendicular to the direction of bulk axial load. Abbasi and Majzooobi³⁴ found that the initial crack angle relative to contact surface was 38-50 degrees for the constant contact load, 49 to 57 degrees for in-phase cyclic contact load and 65 to 70 degrees for out-of-phase conditions. A typical crack growth path can be seen in a specimen fractured by fretting fatigue and shown in Figure 19. As it is seen, the dog bone specimen is broken from the two opposite locations where the fretting pads touch the specimen surface.

Abbasi and Majzooobi³⁴ conducted a series of experiments to assess the effects of phase change between axial and contact loads on crack initiation and propagation lives. They employed a FE model based on Linear Fracture Mechanics (LEFM) to estimate the crack propagation lifetime. A symmetric quarter FE model of the fretting fatigue configuration including load and boundary conditions is shown in Figure 20. A detailed description of the FE simulation can be found in.³⁴

The results showed that while the phase difference had detrimental effect on crack propagation life, it was beneficial to crack initiation life. They found that depending on stress level, crack propagation lifetime was reduced by around 12% to 16% and 26% to 39%, for phase differences of $\phi = 90^\circ$ and $\phi = 180^\circ$, respectively. The results also indicated that for all axial stresses, out-of-phase loading condition significantly

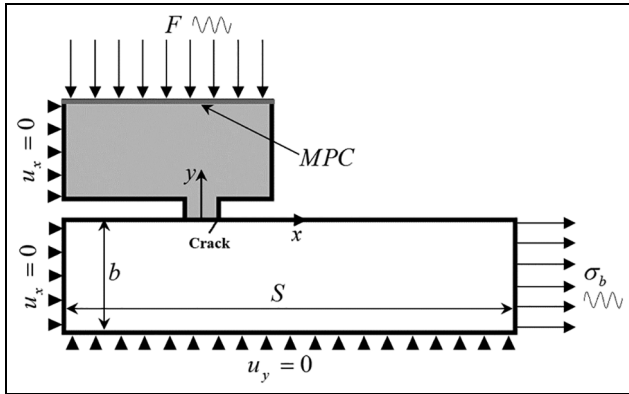


Figure 20. A symmetric quarter FE model of fretting fatigue configuration.

improved the crack initiation lifetime, compared to in-phase loading condition. Depending on stress level, the improvement was about 30% to 57% and 66% to 101%, for phase differences of $\phi = 90^\circ$ and $\phi = 180^\circ$, respectively.

Effect of peak pressure. The effects of contact pressure on fretting fatigue behavior of Al7075-T6 under cyclic contact loading for contact load frequency of $f = 10\text{ Hz}$ was studied by Abbasi and Majzooobi.³⁵ The results showed that, fretting fatigue life declined monotonically with the increase in contact load.³⁵ Figure 21 indicates the variation of predicted crack initiation and crack propagation lifetimes (N_i, N_p) versus the applied axial stresses for different contact loads. In order to estimate the fretting fatigue crack initiation life, they subtracted the fretting fatigue crack propagation life predicted by the FE model, from the total fretting fatigue life obtained in the experiments. Figure 21 Show that with increase the contact and axial loads, the crack initiation portion decreases and as expected the crack propagation portion increases. They concluded that the reduction in crack initiation lifetime which is strongly controlled by the contact stresses is the main factor for decreasing the fretting fatigue life with increasing the contact load.

Effect of contact geometry. Fretting fatigue behavior of Al7075-T6 under cyclic contact loading for different contact geometries was studied by experiment and numerical simulation.³⁶ Two contact geometries including flat-on-flat (FP) and cylinder-on-flat (CP) were considered in the investigation. The frequency of contact load was set to be $f = 10\text{ Hz}$. Figure 22 illustrates variations of predicted crack initiation and crack propagation lives (N_i, N_p) versus the working stresses, S , for both the flat-on-flat and cylinder-on-flat contact types with different pad widths. FPx and CPx

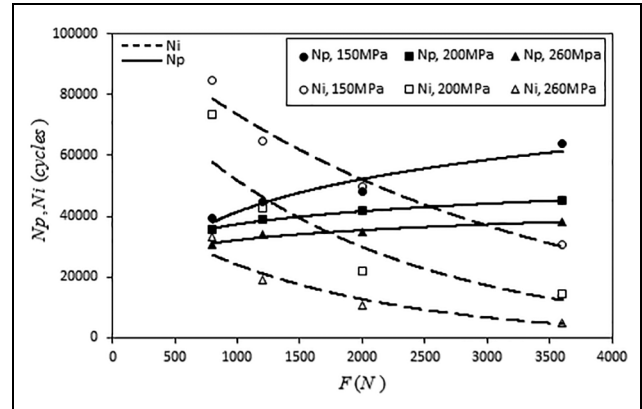


Figure 21. Variation of crack initiation and crack propagation lives versus contact load at different working stresses.

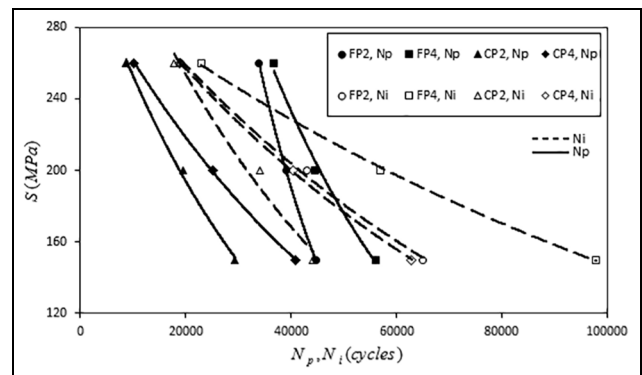


Figure 22. Variation of $S-N_i$ and $S-N_p$ curves for both the flat-on-flat (FP) and cylinder-on-flat (CP) contacts.

demonstrates a pad width of xmm for flat and cylindrical contact type respectively.

From Figure 22 it is seen that, for both of the flat and cylindrical contacts fretting fatigue life improves by increasing the pad width.³⁶ For cylinder-on-flat geometry, both of the crack initiation and crack propagation lives (N_i, N_p) are significantly shorter than those for flat-on-flat geometries. However, the fretting fatigue life is dominated by the crack initiation phase for the cylindrical contact configuration. Majzooobi and Abbasi³⁶ concluded that the main reasons for reduction of fretting fatigue life in cylinder-on-flat configuration, can be attributed to (i) decrease in crack initiation lifetime due to increase in contact stresses that in turn is due to smaller contact width compared to flat-on-flat configuration and (ii) decrease in crack propagation life due to increase in SIFs.

Effect of shot peening. Shot-peening, is a common used cold-working treatment to enhance fatigue resistance of material in aerospace industries. The influence of shot-

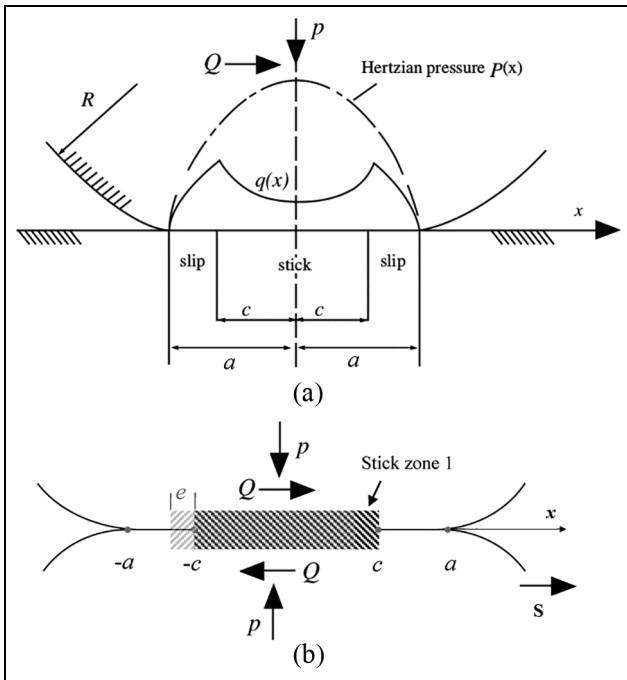


Figure 23. Schematic view of fretting fatigue contact: (a) The stress distribution on the contact area and (b) effect of bulk axial load, S .²⁴

peening treatment on fretting fatigue behavior of material have broadly been studied under constant normal contact load over the past decades. However, less attention has been paid to its effects on fretting fatigue when the contact load is cyclic. A recent study by Majzoubi and Abbasi³⁷ showed that while variable normal contact loading declines the fretting fatigue life remarkably, shot-peening can improve it significantly in both high-cycle and low-cycle fatigue regimes. Depending on the axial stress level, the enhancement was about 89% to 207% and 26% to 143%, for the constant and cyclic contact loading, respectively. It was found that the residual stress induced by shot-peening led to an improvement factor of about 1.44 to 2.62 and 1.16 to 2.10 (depending on the axial stress level) in crack initiation and crack propagation lives, respectively. From their study two effects were found to be responsible to control fretting fatigue life of the shot-peened specimens under cyclic contact loading: compressive residual stress and surface roughening. It was demonstrated that, the beneficial effects of compressive residual stresses dominate over the detrimental effect of surface roughening, and hence resulting in fretting fatigue life enhancement.

Effect of temperature. Temperature is one of the most key factors affecting fretting fatigue behavior of material. The disk slot and blade attachment in the gas

turbine engines are sensitive to fretting fatigue where the operating temperature can be as high as 260°C.⁶⁰ In order to improve the performance of aero-engines under this harmful condition, intensive efforts have been devoted during the past two decades.

Most of the investigations have shown that fretting fatigue lifetime under constant contact loading of the components subjected to high temperatures is much shorter.^{60,61} However, unless the results given by Abbasi and Majzoubi³⁸ no report on the effects of temperature on fretting fatigue behavior of material under cyclic contact loading can be found in the literature. Abbasi and Majzoubi³⁸ showed that the effect of elevated temperature on fretting fatigue behaviour in cyclic contact loading condition was more damaging than that in constant contact loading. The result showed that at higher axial loads where the fretting fatigue life is dominated by the crack propagation portion the detrimental influence of elevated temperature was more profound for crack initiation lifetime. In contrast, at lower axial loads where the life is dominated by the crack initiation portion the adverse influence of elevated temperature was more profound for crack propagation lifetime.

Contact mechanic in cyclic loading

The Hertzian contact geometry is the most popular fretting fatigue contact geometry. Hertzian contact refers to either a cylindrical line contact or a spherical point contact. Formulations of the Hertzian contacts have been accomplished by Hertz,⁶² Mindlin⁶³ and Nowell and Hills.²⁴ Although Hertzian contact mechanics is useful, fretting fatigue occurs due to the combined normal and axial loadings. The application of axial loads does not greatly affect the distribution of normal stress within the contact region. The effect on shear stress is rather more significant. A point within the contact region will experience slip if the product of normal stress at the point and coefficient of friction is equal to the shear stress at that point, $\sigma = \mu\tau$ according to the Coulomb rule. It is therefore, possible for partial slip conditions to occur within a contact area as shown in Figure 23(a). The figure indicates that under the application of normal contact load and shear load alone the partial slip regime, the shear stress distribution consists of a central stick zone and one slip zone on each side of the stick area. The presence of a bulk axial load, S , has the effect of shifting the central stick region, e , towards the rear of the contact depending on the eccentricity level as shown in Figure 23(b). This implies that the center of the contact area can vary between $x = e - c$ and $x = e + c$.

Most of the partial slip approaches which have been reported in the literature use a method in which the

shear stress distribution is assumed as the sum of shear stress due to sliding and a corrected term. Cattaneo⁶⁴ presented the first approach in which at the beginning a classical Hertzian contact is loaded by a contact load P along with application of a tangential load Q . The resulting tangential forces can be considered as the sum of the traction over a smaller central stick zone and a full slip tractions distributed over the entire contact area. Mindlin and Deresiewicz⁶⁵⁻⁶⁷ considered a range of cases where load is applied incrementally and showed that full stick regime would takes place if the increments ΔQ , ΔP in tangential and normal forces, respectively, satisfy the condition $\Delta Q < f\Delta P$, where f is the coefficient of friction. Major progress in the solution of problems involving time-dependent normal and shear loads and exploring fretting fatigue behavior time-histories, was made by Putignano et al.⁶⁸ However, this progress was confined to steady state P-Q problems in which the stick area of the contact interface was symmetrically developed and only its extent as a function of time was to be found.

Barber et al.⁶⁸ have fully described the approach for computation of tangential load distribution during the steady state loading. This approach is briefly discussed in this section. Suppose that the normal contact and tangential tractions are defined by:

$$P(t) = P_0 + P_1 \cos(\omega t) \quad (2)$$

$$Q(t) = Q_0 + Q_1 \cos(\omega t + \phi) \quad (3)$$

Where t is time and ϕ is a relative phase angle between P and Q . As shown in Figure 24, this defines an ellipse in PQ -space. In the absence of gross sliding, the cyclic loading must vary between $Q = +fP$ and $Q = -fP$. If loading begins at $P = Q = 0$, some initial loading phase as OA in Figure 24 is required to reach the cyclic state. Nevertheless, Barber et al.⁶⁹ has shown that this initial phase influences the tangential tractions in the stick area but has no influence on the frictional dissipation process in steady-state contact problems.

The cycle consists of four segments BC , CH , HE , EB , where the points B , C , H , E are defined in Figure 25. As it is seen from the figure, the ellipse is traversed in the anticlockwise direction and dashed lines defining the points all have slope of $\pm f$. Over the path BC , the contact area is growing ($\frac{dP}{dt} > 0$) and $\frac{d|Q|}{dP} < f$, satisfying the condition for the newly developed contact area to remain stuck. Over the path CE there is a growing forward slip zone to point H where the process change so that a line of slope f cuts the opposite side of the ellipse at point B . The system then sticks at point E , after which a reverse slip area grows from the outside of the contact region until once again the system sticks at point B .⁶⁹ In the special case, For $\phi = 0$, the ellipse in Figure 25 turns into a straight line and t in equations (2) and (3) can be eliminated to obtain:

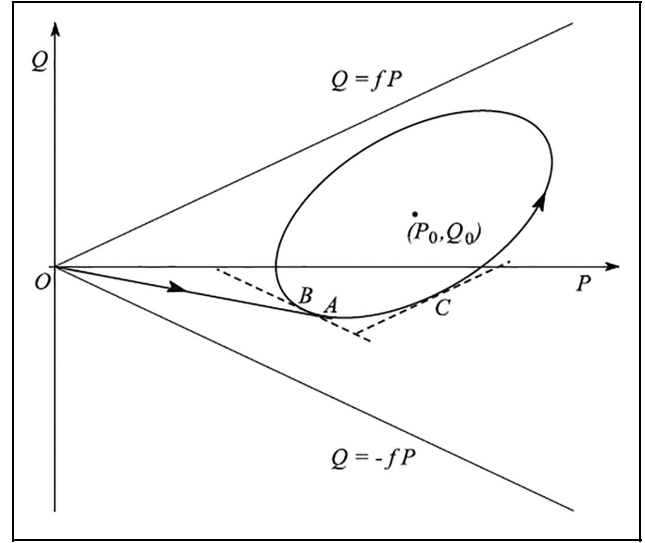


Figure 24. PQ-space for cyclic loading.⁶⁸

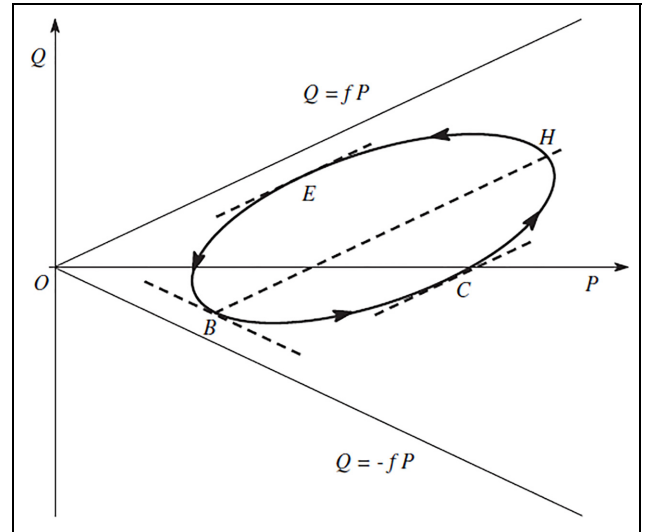


Figure 25. Periodic loading cycle including four segments of the ellipse in PQ -space.⁶⁹

$$Q = \frac{Q_1(P - P_0)}{P_1} \quad (4)$$

If $Q_1 < fP_1$, no slip takes place during loading or unloading and the frictional dissipation is zero. For $|Q_1| > fP_1$, slip will take place during both loading segments. Assume that $Q_1 > 0$ and denote the end points of the line as $A(Q_1, P_1)$ and $B(Q_2, P_2)$ as shown in Figure 26. The trajectory would consist of a straight line from the origin to some point (P_0, Q_0) followed by reciprocating behavior between points A and B whose separation from the mean state is $(\pm \Delta P/2, \Delta Q/2)$. The solution was inherently limited to contacts which are geometrically symmetrical with respect to their

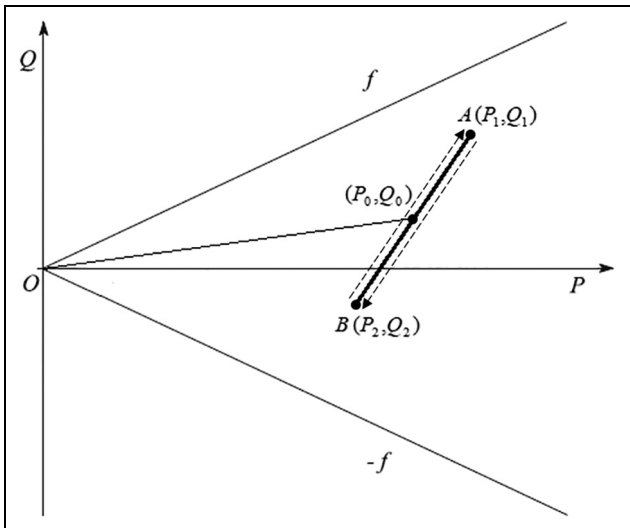


Figure 26. Two dimensional load space for phase angle $\phi = 0$.

centerlines, which is usually not restrictive.⁶⁹ The slip direction is reversed at each edge of the contact zone when going from point A to point B of the loading cycle compared with going from B to A , and the slip regions achieve their maximum extent just before the end points of the loading trajectory are reached. Since we know that the parameter Q_0 does not affect the cyclic dissipation it is possible set $Q_0 = 0$ without loss of generality.⁶⁹ This procedure can be used in a range of applications such as the dovetail root joints of gas turbine. The procedure is easy to implement but the problem with it is that in dovetail and other contact problems, the point of application of the normal load is eccentric with respect to the centerline of the contact and changes with time. Thus, a moment, M , develops at the contact between the disk and the dovetail. More recently, Anderson et al.⁷⁰ provided an applicable method to find the size of the stick zone for a general half-plane contact problem subjected to a constant and cyclic set of loads. This approach can be used only in the steady state condition and is suitable when the axial load is not so high to reverse the sense of slip at a contact edges.

Over the past recent years, several investigations^{71–74} have been focused on the “fatigue aspect” particularly to study the relation between fracture mechanics and the size of contact area. This relation was recognized by a model of fretting known as the “Crack Analogue” (CA)⁷¹ which simplifies the shear traction distribution in contact problems. Recently Ciavarella et al.⁴¹ proposed a modification of the CA model for cyclic contact fretting fatigue problems. This approach was first suggested by Giannakopoulos et al.⁷¹ They suggested that the stress field created near the contact zone is similar to that created ahead of a sharp crack. Thus, using CA model it might be possible to predict the behavior

of the contact problem by finding an equivalent crack. In the so-called CA model the fretting fatigue contact is approximated by an external crack under oscillating mode II. Since in Hertzian contacts the contact pressure goes to zero at the edges (non-singular shear traction distribution), the CA model is suitable if: (i) there is adhesive friction meaning that the coefficient of friction is high enough to create full-stick regime and (ii) the contact is sharp enough. Although the tribological features of the contact and localized surface damages are not considered in their model, the model is just a first effort which can be employed along with the multiaxial fatigue criteria’s for the fatigue design and final assessment in the presence of stress gradients. Furthermore, the theoretical formulation of the V-notched specimen should be extended to the fretting contact problems for making a comparison with the results provided by Ciavarella et al.⁴¹

Conclusion and future directions

The literature review of the studies related to fretting fatigue under cyclic contact loading reveals that there has been substantial breakthrough in this area over the past decade. However, still many aspects of the subjects have remained to be solved. It is well known that complex cyclic contact loading have a detrimental effect of fretting fatigue behavior, reducing the fatigue life with respect to that for the constant contact loading conditions. The broad experimental investigations carried out by Abbasi and Majzoubi^{32–38} can assist to understand the fretting fatigue response of material under complex cyclic contact loading. However as stated by Ciavarella⁷⁵ it is very surprising that slip has occurred under proportional loading. He then concludes that contact mechanics looks unable to explain this basic point that its basics may not be applicable to the case where the frequencies of the contact and the other loads are different.

Recently studies of the contact mechanics under cyclic contact loading has made progress. Contact mechanic can predict the stress distribution within the contact region for idealized geometries and simplified loading. The situations where the loading is cyclic are more complex and often require the consideration of complex multiaxial loading, variable coefficient of friction, frequency effects and complex slip amplitude history. Generally, surfaces will assumed to be perfect and local asperity effects will be taken into account via the application of the coefficient of friction. This simplification leads to ignore the effects of debris build up due to the wear and oxidation especially in cyclic contact loading where the contact load is released periodically exposing the contact region to more oxidation. What the researches have done until now are only a building

block and requires supplementary studies to be performed in future in order to eliminate the shortcomings of the existing tools and approaches, thus further investigation in both analytical and experimental are needed for fretting fatigue phenomenon with cyclic contact loading to be understood adequately.

As a suggestion for future works, by developing a test-rig equipped with suitable sophisticated measuring instruments, more documentary and realistic discussions can be provided by measuring the extent of oxide debris, exposure time to the oxygen, contact interface temperature rise, wear rate, slip amplitude and possible change in the coefficient of friction. In this study, the authors pointed out a need for new fatigue and contact mechanics models by revealing the aforementioned missing parameters. Damage parameters are still being assessed within the research community and it will be useful for a general consensus to be reached on what parameters most effectively model fretting fatigue and the conditions of their use. It is clear that any new parameter attempting to predict fretting fatigue under cyclic loading must use the ideology of damage accumulation over the contact region incorporating load interactions, wear and debris effects, frequency and changing the friction coefficient during cycling. Furthermore, Coulomb friction assumes that the value of friction coefficient remains constant where in reality it is likely to depend on local conditions such as asperities, sliding velocity, slip displacement, temperature, frequency and pressure. However, more complicated friction laws can lead to complex non-linear dynamic effects.⁷⁶ The role of a different static and dynamic coefficient of friction also compels an energetic interpretation of friction.⁷⁷

Finally, in the context of fretting fatigue with cyclic contact loading there are still many latent aspects which must be clarified in future. Some of the way has been travelled but there is still a long way ahead to go.


Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

F Abbasi  <https://orcid.org/0000-0003-2149-2000>

Reference

- Lindley TC. Fretting fatigue in engineering alloys. *J Fatigue* 1997; 19: 39–49.
- Zhang HY, Liu JX and Zuo ZX. Investigation into the effects of tangential force on fretting fatigue based on XFEM. *J Tribol Int* 2016; 99: 23–28.
- Qingming D, Nadeem AB, Xiaochun Y, et al. The effect of a critical micro-void defect on fretting fatigue crack initiation in heterogeneous material using a multiscale approach. *J Tribol Int* 2020; 141: 39–49.
- Llavori I, Zabala A, Aginagalde A, et al. Critical analysis of coefficient of friction derivation methods for fretting under gross slip regime. *J Tribol Int* 2019; 143: 105988.
- Navarro C, Vázquez J and Domínguez J. Nucleation and early crack path in fretting fatigue. *J Fatigue* 2017; 100: 602–610.
- Zabala A, Infante-García D, Giner E, et al. On the use of the theory of critical distances with mesh control for fretting fatigue lifetime assessment. *J Tribol Int* 2020; 142: 105985.
- Vázquez J, Navarro C and Domínguez J. Analysis of fretting fatigue initial crack path in Al7075-T651 using cylindrical contact. *J Tribol Int* 2017; 108: 87–94.
- Pereira K and Magd AW. Fretting fatigue lifetime estimation using a cyclic cohesive zone model. *J Tribol Int* 2020; 141: 105899.
- Navarro C, Vázquez J and Domínguez J. A general model to estimate life in notches and fretting fatigue. *J Eng Frac Mech* 2011; 78: 1590–1601.
- Nadeem AB and Magd AW. Fretting fatigue crack nucleation: a review. *J Tribol Int* 2018; 121: 121–138.
- Farrahi GH, Minaii K and Bahai H. Fretting fatigue behavior of 316L stainless steel under combined loading conditions. *J Fatigue* 2019; 128: 105206.
- Wang S and Magd AW. Effect of loading conditions in fretting fatigue on wear characteristics. In: *Proceedings of the 13th international conference on damage assessment of structures*, Porto, Portugal, 9–10 July 2019, pp. 659–665. Singapore: Springer.
- Ciavarella M, Papangelo A and Barber JR. Effect of wear on the evolution of contact pressure at a bimaterial sliding interface. *J Tribol Lett* 2020; 68:27.
- Araújo JA, Castro FC, Matos IM, et al. Life prediction in multiaxial high cycle fretting fatigue. *J Fatigue* 2020; 134:105504.
- Vakis AI, Yastrebov VA, Scheibert J, et al. Modeling and simulation in tribology across scales: an overview. *J Tribol Int* 2018; 125:169–199.
- Pompanon F, Laporte J, Fouvry S, et al. Normal force and displacement amplitude influences on silver-plated electrical contacts subjected to fretting wear: a basic friction energy – contact compliance formulation. *J Wear* 2019; 426–427: 652–661.
- De Pannemaecker A, Fouvry S, Buffiere JY, et al. Modelling the fretting fatigue crack growth: From short crack correction strategies to microstructural approaches. *J Fatigue* 2018; 117: 75–89.
- Montebello C, Pommier S, Demmou K, et al. Analysis of the stress gradient effect in fretting-fatigue through non-local intensity factors. *J Fatigue* 2016; 82: 188–198.
- Barbara F, Araújo JA, Pommier S, et al. Life of a Ti–6Al–4V alloy under fretting fatigue: Study of new nonlocal parameters. *J Tribol Int* 2017; 108: 23–31.

20. Gowda BMA, Yeshovanth HR and Siddaraju C. Investigation and efficient modeling of an dovetail attachment in aero-engine. *J Proc Mat Sci* 2014; 5: 1873–1879.
21. Matlik JF, Farris TN, Haake FK, et al. High-frequency, high-temperature fretting-fatigue experiments, *J Wear* 2006; 261: 1367–1382.
22. Vingsbo O and Soderberg S. On fretting maps. *J Wear* 1988, 126:131–147.
23. Hojjati-Talemi R, Wahab MA and De Baets P. Finite element simulation of phase difference effects on fretting fatigue crack nucleation behaviour. *Proc Inst Mech Eng, Part J: J Eng Tribol* 2014; 228: 470–479.
24. Hills DA and Nowell D. Mechanics of fretting fatigue. *J Wear* 1994; 175:107–113.
25. Madge JJ, Leen SB, McColl IR, et al. Contact-evolution based prediction of fretting fatigue life: effect of slip amplitude. *J Wear* 2007; 262: 1159–1170.
26. Pearson SR and Shipway PH. Is the wear coefficient dependent upon slip amplitude in fretting? Vingsbo and Söderberg revisited. *J Wear* 2015; 330: 93–102.
27. Nesládek M, Španiel M, Jurenka J, et al. Fretting fatigue-experimental and numerical approaches. *J Fatigue* 2012; 44: 61–73.
28. Ciavarella M. The generalized Cattaneo partial slip plane contact problem. I-Theory. *J Solids Struct* 1998; 35: 2349–2362.
29. Ruiz C, Bodington P and Chen K. An investigation of fatigue and fretting in a dovetail joint. *J Exp Mech* 1984; 24: 208–217.
30. Miner MA. Cumulative damage in fatigue. *J Appl Mech* 1945; 12: 159–164.
31. Mall S, Jutte AJ, Fuchs SP, et al. Investigation into variable contact load effects on fretting fatigue behavior of Ti-6Al-4V. In: *XXI ICTAM*, Warsaw, Poland 15–21 August 2004. Dordrecht, The Netherlands: Springer.
32. Abbasi F, Majzoobi GH and Barjesteh MM. Developing a new experimental set up to study fretting fatigue behavior under cyclic contact loading. *Proc IMechE Part J: J Eng Tribol* 2017; 232: 837–850.
33. Majzoobi GH and Abbasi F. An investigation into the effect of normal load frequency on fretting fatigue behavior of Al7075-T6. *J Tribol Trans* 2018; 61: 547–559.
34. Abbasi F and Majzoobi GH. Effect of out-of-phase loading on fretting fatigue response of Al7075-T6 under cyclic normal loading using a new testing apparatus. *J Eng Fract Mech* 2018; 188: 93–111.
35. Abbasi F and Majzoobi GH. Effect of contact pressure on fretting fatigue behavior under cyclic contact loading. *J Surf Rev Lett* 2018; 25:1–14.
36. Majzoobi GH and Abbasi F. On the effect of contact geometry on fretting fatigue life under cyclic contact loading. *J Tribol Lett* 2017; 65: 125.
37. Majzoobi GH and Abbasi F. On the effect of shot-peening on fretting fatigue of Al7075-T6 under cyclic normal contact loading. *J Surf Coat Tech* 2017; 328: 292–303.
38. Abbasi F and Majzoobi GH. An investigation into the effect of elevated temperatures on fretting fatigue response under cyclic normal contact loading. *J Theor Appl Fract Mech* 2018; 93: 144–154.
39. Abbasi F. Some aspects of fretting fatigue under complex cyclic contact load condition. *J Mech Eng Res* 2018; 1: 16–21.
40. Xin L, Jianwei Y, Meihong L, et al. An investigation on fretting fatigue mechanism under complex cyclic loading conditions. *J Fatigue* 2016; 88: 227–235.
41. Ciavarella M and Berto F. A simplified extension of the Crack Analogue model for fretting fatigue with varying normal load. *J Theor Appl Fract Mech* 2017; 91: 37–43.
42. Huq MZ, Butaye C and Celis JP. An innovative system for laboratory fretting wear testing under oscillating normal force. *J Mater Res* 2000; 15: 1591–1599.
43. Huq MZ and Celis J. Fretting fatigue in alumina tested under oscillating normal load. *J Am Ceram Soc* 2002; 85: 986–988.
44. Lykins CD, Mall S and Jain VK. A shear stress based parameter for fretting fatigue crack initiation. *J Fat Fract Eng Mater Struct* 2001; 24: 461–473.
45. Nowell D. Recent developments in the understanding of fretting fatigue. In: *15th European conference in fracture-advanced fracture mechanics for life and safety assessments*, Stockholm, Sweden, 11–13 August 2004. Sweden: Lund University Publications.
46. Iyer K. Peak contact pressure, cyclic stress amplitudes, contact semi-width and slip amplitude: relative effects on fretting fatigue life. *J Fatigue* 2001; 23(3):193–206.
47. Jutte AJ. *Effect of a variable contact load on fretting fatigue behavior of Ti-6Al-4V*. MSc Thesis, Air Force Institute of Technology. USA, 2004.
48. Rajasekaran R and Nowell D. Fretting fatigue in dovetail blade roots: experiment and analysis. *J Tribol Int* 2006; 39: 1277–1285.
49. De Pauw J, De Baets P, De Waele W, et al. Design of a fretting fatigue test rig with compliant springs. In: *Sustainable Construction and Design (SCAD-2013)*, Ghent, Belgium, 20–21 February 2013. Zwijnaarde, Belgium: Laboratory Soete, Ghent University Technologiepark.
50. Dobromirski J. Variables of fretting process: are there 50 of them? *ASTM Int* 1992; 1159: 60.
51. Antoniou RA and Radtke TC. Mechanisms of fretting fatigue of titanium alloys. *J Mater Sci Eng* 1997; A237: 229–240.
52. Brown SR. Materials evaluation under fretting conditions. *ASTM Int* 1982; 780.
53. Kennedy P, Peterson MB and Stallings L. An evaluation of fretting at small slip amplitude. *ASTM Int* 1982; 780: 30–48.
54. Iyer K and Mall S. Effects of cyclic frequency and contact pressure on fretting fatigue under two-level block loading. *J Fat Fract Eng Mater Struct* 2000; 23: 335–346.
55. Lykins CD, Mall S and Jain VK. Prediction of fretting fatigue crack initiation based on the plane of maximum shear stress amplitude. In: *41st Structures, structural dynamics, and materials conference and exhibit*, Atlanta, GA, USA, 3–6 April 2000. American Inst. of Aeronautics and Astronautics.
56. Lee C. *Effects of variable contact load on fretting fatigue behavior of shot-peened and un-peened Titanium alloy*. MSc Thesis, Air Force Institute of Technology, USA, 2004.

57. Lee H and Mall S. Investigation into tangential force and axial stress effects on fretting fatigue behavior. *J Eng Mater Tech* 2006; 128: 202–209.
58. Almajali M. *Effects of phase difference between axial and contact loads on fretting fatigue behavior of titanium alloy*. MSc Thesis, Air Force Institute of Technology, USA, 2006.
59. Al-Noaimi A. *Investigation into fretting fatigue under cyclic contact load and in conjunction with plain fatigue of titanium alloy*. MSc Thesis, Air Force Institute of Technology, USA, 2008.
60. Jin O, Mall S and Sahan O. Fretting fatigue behavior of Ti-6Al-4V at elevated temperature. *J Fatigue* 2005; 27: 395–401.
61. Lee H and Mall S. Stress relaxation behaviour of shot-peened Ti-6Al-4V under fretting fatigue at elevated temperature. *J Mater Sci Eng* 2004; 366: 412–420.
62. Hertz H. On the contact of elastic solids. *J reine angew Math* 1881; 92: 156–171.
63. Mindlin R. Compliance of elastic bodies in contact. *AASME Trans J Appl Mech* 1949; 16: 259–268.
64. Cattaneo C. Sul contatto di due corpi elastici: distribuzione locale degli sforzi. *Rend Dell Accad Naz Dei Lincei* 1938; 27: 342–348.
65. Mindlin RD and Deresiewicz H. Elastic spheres in contact under varying oblique forces. *ASME J Appl Mech* 1953; 75: 327–344.
66. Jager J. Half-planes without coupling under contact loading. *J Arch Appl Mech* 1997; 67: 247–259.
67. Jager J. A new principle in contact mechanics. *J Tribology* 1998; 120: 677–684.
68. Barber J, Davies M and Hills D. Frictional elastic contact with periodic loading. *J Solids Struct* 2011; 48: 2041–2047.
69. Putignano C, Ciavarella M and Barber JR. Frictional energy dissipation in contact of nominally flat rough surfaces under harmonically varying loads. *J Mech Phys Solids* 2011; 59: 2442–2454.
70. Andresen H, Hills DA, Barber JR, et al. Steady state cyclic behaviour of a half-plane contact in partial slip subject to varying normal load, moment, shear load, and moderate differential bulk tension. *J Solids Struct* 2020; 182–183: 156–161.
71. Giannakopoulos AE, Lindley TC and Suresh S. Aspects of equivalence between contact mechanics and fracture mechanics: theoretical connections and a life prediction methodology for fretting-fatigue. *J Acta Mater* 1998; 46: 2955–2968.
72. Davies M, Barber JR and Hills DA. Energy dissipation in a frictional incomplete contact with varying normal load. *J Mech Sci* 2012; 55: 13–21.
73. Pook LP, Campagnolo A, Berto F, et al. Coupled fracture mode of a cracked plate under anti-plane loading. *J Eng Fract Mech* 2015; 134: 391–403.
74. Ciavarella M. Some observations on the CLNA model in fretting fatigue. *J Tribol int* 2006; 39: 1142–1148.
75. Ciavarella M. Discussion on “effect of out-of-phase loading on fretting fatigue response of Al7075-T6 under cyclic normal loading using a new testing apparatus by F. Abbasi and G.H. Majzoobi”. *J Eng Fract Mech* 2018; 192: 205–209.
76. Papangelo A, Ciavarella M and Hoffmann N. Subcritical bifurcation in a self-excited single-degree-of-freedom system with velocity weakening–strengthening friction law: analytical results and comparison with experiments. *J Nonl Dyn* 2017; 90: 2037–2046.
77. Papangelo A and Ciavarella M. Cattaneo–Mindlin plane problem with Griffith friction. *J Wear* 2015; 342: 398–407.