INVENTORY ROUTING PROBLEM WITH STOCHASTIC DEMAND AND LEAD TIME

DOCTORATE THESIS
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For my family.
There is a woman at the beginning of all great things.

— Alphonse de Lamartine

To God for giving me the opportunity to develop my doctoral thesis.

Dedicated to my loving wife Stella and my loving daughter Mariana. Whose love and confidence is a constant source of inspiration and encouragement.

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ABSTRACT

In the supply chain, the integration of the different processes is critical to obtain high levels of coordination. Inventory control and its distribution are two of these processes whose coordination have been demonstrated by researchers as key in order to gain efficiency and effectiveness. They affect the synchronization of the supply chain management. With the intention to contribute to the integration of these processes and improve the problems of demand variability, we propose an integration of operations research area and the help of metaheuristics in a multi-objective approach. The expected results are to reduce the costs associated with inventory and its distribution, as well as to reduce the uncertainty in making decisions based on demand.

This thesis presents methods for obtaining and analyzing near optimally solutions for dynamic and stochastic inventory-routing problems. The methods include retailers selection and clustering methods, algorithms and experiments on benchmark instances. We focus on problems with one and several suppliers that serve several dispersal geographically retailers.

The thesis contains four parts. In Part I, we focus on the literature review. We first provide an overview of the literature on problems related to the coordination of the inventory and its distribution. Then we make a point in four elements: information management, inventory policies, stochastic demand and optimization methods. Also, we provide a scientometric analysis of the documentation collected in the last ten years. We provide a thorough review of papers working with dynamic and stochastic demand. The contributions of this part are i) the review of papers working with stochastic demand and stochastic lead times focusing on its stochastic and multi-depot aspects, ii) identify critical factors for the performance of many logistics activities and industries, iii) have shown that studying the behavior of the demand and the lead time are essential in order to achieve a useful representation of the system to take proper decisions and iv) provide the trends and patterns in the research in IRP problems.

In Part II, we focus on the methodology of the research and of development. We first introduce the problem, state of the science, the gaps in the literature, variables under study, the instruments applied and assumptions. The development methodology is presented by a general model to address this type of research proposed in this thesis. Here, the general development process, decomposition of the problem and how the possible solutions are explained.. The importance of the this chapter is provided an effective way to face IRP problems.

In Part III, the foundations in formulations for IRP problems are proposed. We begin with the formulation of the TSP problems with variants for one and many suppliers, likewise for VRP and IRP problems. The contributions of the model presented here aim identifying the variables and mathematical models frequently used to deal with these problems.

In Part IV, we perform a single criteria objective and multi-criteria analysis of the solutions for one and many suppliers instances. Our methods yield significant improvements over a competing algorithm. Our contributions are i) propose three new customer selection methods for a dynamic and stochastic inventory-routing

problem, ii) perform a multi-criteria analysis of the solutions, comparing distribution versus inventory management, iii) perform a single criteria objective experiment on benchmark instances from the literature.

RESUMEN

En la cadena de suministro, la integración de los diferentes procesos que la conforman, es fundamental para obtener altos niveles de coordinación. El control del inventario y su distribución son dos de estos procesos, cuya coordinación ha sido demostrada por los investigadores como clave para lograr mejoras en eficiencia y efectividad. Estos a su vez, afectan la sincronización y la administración de la cadena de suministro. Con el propósito de contribuir en la integración de éstos procesos y mejorar los problemas derivados de la variabilidad de la demanda, se propone usar los fundamentos del área de investigación de operaciones y la ayuda de metaheurísticas en un enfoque multi-obejtivo. Los resultados esperados son reducir los costos asociados a los procesos de inventario y distribución, así como también reducir la incertidumbre en la toma de decisiones a partir de la demanda.

Ésta tesis presenta métodos para el análisis y obtención de soluciones cercanas a las óptimas para problemas de inventario y routeo, dinámico y estocástico. Los métodos incluyen selección de retailers y métodos de clustering, algoritmos y experimentos en instancias de prueba disponibles en la literatura. Se hace énfasis en instancias de un solo proveedor y varios proveedores que sirven varios retailers distribuidos geográficamente.

La tesis está organizada en cuatro partes. En la Parte I, se revisa la literatura, para ello, primero se presentan los problemas relacionados con la coordinación del inventario y su distribución. Ésta revisión resalta cuatro elementos que han sido identificados como claves en la literatura como son: la administración de la información, políticas de inventario, demanda estocástica y métodos de optimización. Luego, se presenta un análisis cienciometrico de la literatura encontrada en los últimos 10 años. La revisión de la documentación se realiza de manera exhaustiva trabajando con demanda dinámica y estocástica. Las contribuciones de esta parte son: i) proporcionar una revisión pertinente y actualizada de artículos que emplean demanda estocástica, enfatizando en sus elementos dinámicos y estocásticos, así como también en aspectos que permitan abordar problemas con múltiples depósitos, ii) identificar factores críticos para el desempeño de actividades logísticas, iii) Demostrar que el estudio de la demanda es esencial para lograr una representación útil del sistema, la cual influye en la toma de decisiones y iv) proporcionar tendencias y patrones en la investigación de problemas de IRP.

En la Parte II se aborda la metodología de la investigación y de desarrollo. Primero, se presenta el problema, el estado de la ciencia y los gaps encontrados en la literatura. Luego se identifican las variables de estudio, los instrumentos aplicados y los supuestos utilizados. La metodología de desarrollo es presentada por medio de un modelo general para abordar éste tipo de investigaciones que nosotros proponemos en ésta tesis. Esta metodología aborda aspectos como: el procedimiento general de desarrollo, la descomposición del problema y la forma en que se prueban las posibles soluciones.

En la Parte III, se presentan los fundamentos en la formulación de IRP. Primero se formulan los problemas TSP con variantes para un solo depósito y también para

múltiples depósitos, igualmente se hace para VRP e IRP. La contribución de los modelos presentados son la identificación de las variables y los modelos matemáticos que frecuentemente son usados para tratar con éste tipo de problemas.

En la Parte IV se presentan dos experimentos. El primero para el análisis de instancias con uno sólo depósito y en el segundo para analizar instancias con múltiples depósitos. Los métodos usados producen mejoras sobre resultados obtanidos con algoritmos similares. Las contribuciones de ésta parte son: i) proponer tres nuevos métodos para la selección de retailers para IRP dinámicos y estocásticos, ii) realizar análisis multi-criterio de las soluciones, comparando la distribución con la administración del inventario y iii) realizar análisis de un solo objetivo sobre instancias de pruebas proporcionada por la literatura existente.

PUBLICATIONS

This might come in handy for PhD theses: some chapters, ideas and figures have appeared previously in the following publications:

JOURNAL PAPERS

Raúl F. Roldán, Rosa Basagoiti, and Leandro C. Coelho. "A survey on the inventory routing problem with stochastic lead times and demands." In: Journal of Applied Logic (2016). Article in press. [98]

Raúl F. Roldán, Rosa Basagoiti, and Leandro C. Coelho. "Robustness of inventory replenishment and customer selection policies for the dynamic and stochastic inventory-routing problem." In: Computers & Operations Research (2016). Article in press. [99]

CONFERENCE PAPERS

Raúl F. Roldán, Rosa Basagoiti, and Enrique Onieva. "Framework in The Formulation and Solution of Inventory Routing Problems." In: 15th International Conference on Operational Research, KOI 2014. Croatian Operational Research Society. Osijek, Croatia, 2014. [100]

Raúl F. Roldán, Rosa Basagoiti, and Enrique Onieva. "Inventory routing problem with stochastic demand and lead time: State of the art." In: Advances in Intelligent Systems and Computing 299 (2014), pp. 73–82. [101]

"Teaching of calculus to engineers and physicists,
could be essentially improved
if the nature of heuristic reasoning
were better understood"

George Pólya, How to Solve It: A New Aspect of Mathematical Method

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"La enseñanza del cálculo en los ingenieros y en los físicos,
podría mejorarse esencialmente
si la naturaleza del razonamiento heurístico
se entiende mejor"

George Pólya, How to Solve It: A New Aspect of Mathematical Method

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ACRONYMS

SC Supply Chain

SCM Supply Chain Management

BWE BullWhip Effect

VMI Vendor Managed Inventory

EOQ Economic Order Quantity

MOO Multi-Objective Optimization

TSP Travelling Salesman Problem

mTSP Multi-Travelling Salesman Problem

VRP Vehicle Routing Problem

CVRP Capacitated Vehicle Routing Problem

MDVRP Multi-depot VRP

PVRP Periodic VRP

VRP Multi-depot and Periodic Vehicle Routing Problem

VRPTW Vehicle Routing Problem with Time Windows

MDHFVRP Depot and Heterogeneous Fleet Vehicle Routing Problem

IRP Inventory Routing Problem

DSIRP Dynamic and Stochastic Inventory Routing Problem

GA Genetic Algorithm

TS Tabu Search

SA Simulated Annealing

EA Evolutionary Algorithm

LS Local Search

ANN Artificial Neural Networks

VNS Variable Neighborhood Search

SCM Straight Count Method

APM Author Position Method

NCII Normalized Citation Impact Index

NCIIW Normalized Citation Impact Index weighted

GS Google Scholar

SJR SCImago Journal Rank

SNIP Source Normalized Impact per Paper

IF Impact Factor

5YIF 5-Year Impact Factor

ML Maximum Level

OU Order-Up-to

DF Demand Forecasting

BOF Big Orders First

LSF Lowest Storage Capacity First

REQ Rest Equal Quantity

RW Roulette Wheel

BT Binary Tournament

BD Binary Tournament and Diversity

FQ Fixed Quantity

AD Ahead Demand

IRCSPA Inventory Replenishment and Customer Selection Policies Algorithm

HGANFF Hybrid Genetic Algorithm with Network Flow Fitness

MDDSIRP Multi Depot Dynamic Stochastic Inventory Routing Problem

1 INTRODUCTION

The coordination and the integration of the various components in the Supply Chain (SC) management have become critical in gaining competitive advantage. The price of the a product is the key for the competitiveness, but it is affected by the logistics cost which increases their cost. A great proportion of the logistic costs correspond to the transportation and inventory processes. The inventory represents a proportion of net operating assets of approximately 37% in industry, 62% in distribution and 56% in retail.

In turn, the variability of the demand information affects the integration of the SC. The variability causes efficiency and efficacy losses influencing the decisions related to inventory control. In addition, it is important to note that the inventory control has to balance conflicting objectives due to two main reasons: i) economy of scale and purchasing batch size and ii) uncertainty in offer and demand with production and transportation lead time that inevitably creates the need for safety stock.

Besides that, the current models proposed in the literature are computationally efficient but have some difficulties of implementation in the real world due to their lack of flexibility when incorporating additional constraints. In this context, it is mandatory to establish an optimal policy for the coordination of the flow of goods and services along the SC. This policy is applicable in the process of distribution and inventory in commercial relationships between companies. The overall costs are minimized as well as the uncertainty in the decisions. The decisions to be taken are related to how much to ship, when to ship and how to ship. These decisions are taken in oder to decrease inventory and transportation cost, guaranteeing a certain service level and adjusting to available resources.

Given the complexity of the optimization of the inventory and its distribution, problems generally called as Inventory Routing Problem (IRP), the studies are approached from instances. Models usually include only the variables of interest according to real-world problems. A basic problem to be considered have the following characteristics: the SC contains one or more suppliers of products and several geographically dispersed retailers. The Vendor Managed Inventory (VMI) policy is followed and one only actor is responsible for taking decisions. The product demand is assumed unknown and its is reveled gradually in a given planning horizon. Each retailer has a capability of storage. One or several vehicles are used to transport where its capacity is limited and each vehicle only does one trip by period.

The objective is to minimize the total cost of inventory and its distribution and minimize or eliminate stock-outs in the retailers. The cost is related to the inventory cost in the suppliers, the inventory cost in the retailers and the transportation cost. Thus, it is necessary to determine the inventory level by period for each retailer and to determine strategies that minimize the associated costs for the distribution routes and inventory held.

The performance and coordination and integration of the components of the SC is critical to gain competitive. The logistic costs, in special those related to inventory and its distribution, do not add value to a product, but increase its price and produces losses of market, expensive products, and higher inventory levels than recommended. The most representative processes that add cost in the SC are the inventory and transportation.

In this context, one must establish an optimal policy for the coordination of the flow of goods and services along the SC in commercial relationships between suppliers and companies. The main objective is to coordinate inventory control and its distribution process in the SC, in order to reduce the cost and the uncertainly in the taking decisions based on demand.

The work is divided in four parts. In Part II a global overview of the real situation of the problems related to the coordination of the inventory and its distribution is provided. This part is composed by two chapters. In Chapter 2, a state of the art is provided, emphasizing five aspects:

- Problem, justification and necessity. The problem is presented under of an approach of structure of cost.
- Information management in the SC. Deals with the information as a key element to coordinate processes in the SC. In this context, the BullWhip Effect (BWE) is highlighted as a problem that causes efficiency and efficacy loses and the necessity of centralize the decisions for reducing its variability.
- Inventory policies addresses the decisions that need to be taken when dealing with the inventory and its distribution and as there can be controlled by means of inventory policies
- Demand and lead time modeling under uncertainly deals with the demand and as it is possible through of its estimation and its knowledge to have better possibilities to avoid stock-outs, surplus and to reduce the BWE.
- Optimization methods, here, several methods for inventory optimization and its distribution are showed. This methods take into account near-optimal solutions which are obtained with heuristics and metaheuristics, in especial those bio-inspired used in computer science.

In Chapter 3, a scientometric analysis is presented, empirically studying the evolution of the research on IRP providing a view about groups of researchers, their research productivity and impact, all that providing a better understanding of trends. The chapter shows through of the relevant publications on DSIRP, the analysis of number of publications per category, the analysis of citations per category, groups of researchers, the trends and future work on the field.

In Part II, the methodological aspects are explained by referring two chapters. In Chapter 4, the research methodology is addressed consider 7 aspects which they are listed below:

- The scope and significance of the problem, here the research problem is presented, is delimited and is justified.
- The state of the science relates to objects of the study Inventory Routing Problems and inventory Control. These objects are addressed by means the topics stated in the state of the art.
- Gaps in the literature specifies purpose, questions, hypothesis and contributions of the research.

- Variables under study, here the independents, the dependents and the intervening variables are identified.
- Operationalization of variables shows some metrics to evaluate the obtained solutions.
- Instruments applied and data collection highlights the use of benchmark instances.
- Assumptions. A set of assumptions are defined.

In Chapter 5 the details about the methodology of development are explained by means of:

- Preliminary procedure of development shows the steps carried out to solve the problem
- Initial decomposition to the problem shows the possible phases in that the problem can be divided
- Testing feasible solutions indicates how the possible are the comparisons to validate the performance of the algorithm.

In Part III, the background formulations for IRP is provided, step by step from TSP problems to IRP with periods of time and multi suppliers versions.

Finally, in Part IV, two experiments were performed and two algorithms were designed. In Chapter 7 a heuristic model to solve DSIRP is presented in order to test many policies of reduction of cost in the SC. The results are analyzed taking into account two approaches: with one objective and with two objectives. The tests allow projecting future work. The chapter is divided in the following sections: In Section 7.2 we formally describe the problem. In Section 7.3 we present our solution procedure which includes customer selection, quantities determination, and vehicle routing. In Section 7.4, we present the results of extensive computational experiments and we analyze the trade-off between inventory and transportation costs. We describe how we can identify dominated solutions under a multi-objective optimization approach, and we compare our solutions against the ones from the literature. In Section 7.5 we present our conclusions and findings.

The second experiment in Chapter 8 is addressed. In this chapter a relatively simple but effective hybrid GA to solve the multi supplier DSIRP is explained and evaluated. In terms of reduction of average costs, the results will show a good performer to compare them with a lower and higher boundaries from exact and heuristics derived from literature.

And finally the conclusions and future work are provided.

Part I

LITERATURE REVIEW

2 DYNAMIC AND STOCHASTIC INVENTORY ROUTING PROBLEM (DSIRP)

An article based on this chapter was published for Advances in Intelligent Systems and Computing Volume 299, 2014, Pages 73-82 by Raúl Roldán, Rosa Basagoiti and Enrique Onieva which has the title of Inventory routing problem with stochastic demand and lead time: State of the art ([101]). The most recent version of this article is currently accepted for publication in Journal of Applied Logic by Raúl Roldán, Rosa Basagoiti and Leandro Coelho, which has the title A Survey on the Inventory Routing Problem with Stochastic Lead Times and Demands ([98]).

The integration of the different processes that conform the supply chain (SC) is fundamental to obtain a better coordination level. The inventory control and its distribution, are the processes that researches have found as the key in the loss of efficiency and effectiveness in the field of logistics, affecting so the synchronization in the SC management. In order to analyze the recent developments in the integration of these processes, this paper analyzes the state of the art of the progress in information management in the SC, the relationship of inventory policies and the demand information, modeling demand and use of optimization methods in the search for the appropriate solutions.

With the aim of providing a global overview of the real situation of the problems related to the coordination of the inventory and its distribution, in this chapter a state of the art is provided. We have identified four key elements that should be taken into account to propose alternative solutions, so this study highlights: i) the information management between different actors in the SC, since this determines the evolution and quality of information, ii) inventory policies and their relation to the demand information, in order to properly manage inventory levels, iii) stochastic demand and lead time modeling, to understand and represent their behavior over time and iv) optimization methods for the search of the most appropriate solution.

In this chapter, our goal is to provide an overview of the literature on problems related to the coordination of the inventory and its distribution. We provide a thorough review of papers working with stochastic demand and stochastic lead times, as these are the factors identified as critical for the performance of many logistics activities and industries.

In Section 2.1 the logistic cost and how they increase the price of the products is dealt, difficulties to access for the new products to the markets and the inventory levels increase. Section 2.2 deals with the information as a key element to coordinate process in the SC. In this context, the BWE is highlighted as a problem that causes efficiency and efficacy losses.

Three decisions that have to be taken when dealing with the inventory and its distribution: i) when replenish, ii) how much to replenish and iii) how often the inventory level is reviewed. These decisions can be taken by means of an inventory policy. Thus, Section 2.3 deals with the more important policies in uncertainly demand. These policies can reduce directly the cost in SC, in special in the determination of clients that should be served in every period.

Section 2.4 deals with the demand and lead time. Using the estimation and the knowledge about the demand to have better possibilities to avoid stock-outs, surplus and reduced the BWE. Here it is important to notice that an adequate model of the demand is essential for this purpose. The section describes some models that have been used to understand and explain the nature of the demand.

In Section 2.5 several methods for inventory optimization and its distribution are showed. This methods take into account near-optimal solutions which are obtained with heuristics and metaheuristics, in especial those bio-inspired used in computer science.

In the final Section the conclusions are outlines taking into account all the previous topics.

2.1 PROBLEM, JUSTIFICATION AND NECESSITY

SC is defined by Blanchard [18], as the sequence of events that cover a product's entire life cycle, from conception to consumption and that involves different actors such as suppliers, producers, distributors, transporters and clients among others, all of them involved directly or indirectly in satisfying the request of a final client, "more and more companies become aware of their SC performance, the coordination and the integration of the various components in the SC management have become critical in gaining competitive advantage" [88].

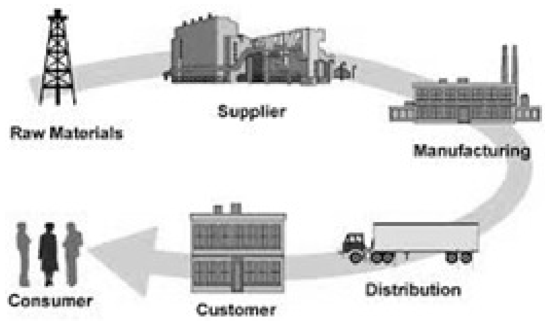


Figure 1: Actors of the supply chain

The study Guasch and Kogan [52] compares the logistics costs of the European Organization for Economic Cooperation (OECD) and Latin America (LAC), in order to find the impact of logistics cost on the price of the product. The authors conclude that the costs associated with the inventory represents a proportion of the price of a product that is in average approximately 19% (see Table 1), in contrast, in Singapore that was ranked No.1 in the logistic performance index 2012, it is only 8.5% (see Figure 3). According with Guasch [51], the reduction in cost logistics can be evidenced by two indicators: i) the increase in the proportion of demand of a product launched on the market and ii) the increasing in the proportion of employment that can be generated for some sectors of the economy (see Table 2). So, the increase in product price obstructs the competitiveness and complicates the maintenance of inventory.

On the other hand, in Guasch [51], the logistics costs are divided in: administrative costs, warehousing, inventory, transportation and licenses. From this study it can be concluded that more than 69% of these costs are directly related to transportation and inventory (see Figure 4). Just the inventory represents a proportion of net operating assets of approximately 37% in industry, 62% in distribution and 56% in retail

Table 1: Proportion of costs associated with the inventory [52]

Element	Average(%)	Ranges(%)
Capital Cost	15,00	8-40
Taxes	1,00	0,35-1,52
Insurance	0,05	0,01-0,25
Obsolescence	1,20	0,5-3
Storage	2,00	0-4
TOTALS	19,25	9-50

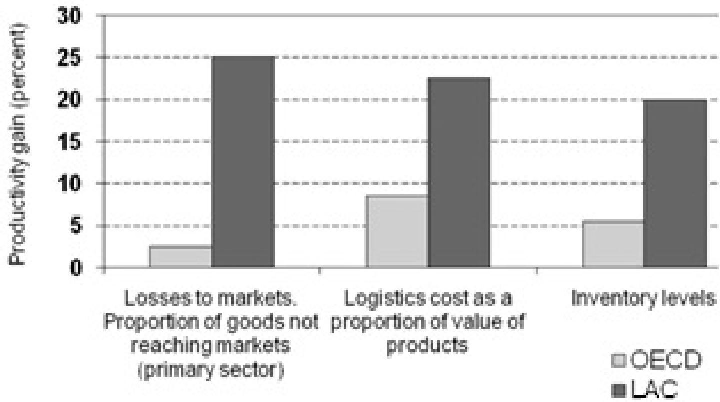


Figure 2: Proportion of productivity gain versus logistics costs [52]

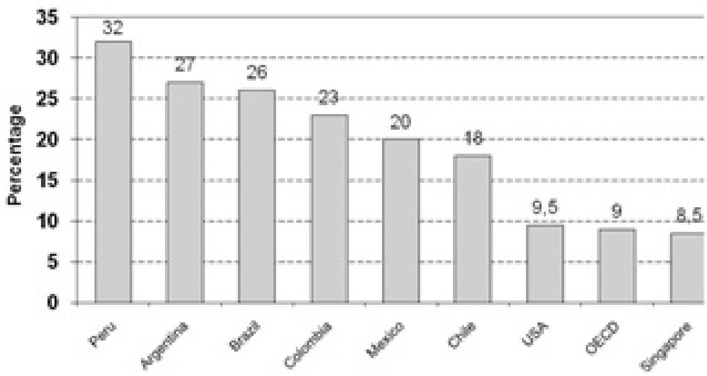


Figure 3: Proportion of the product value that corresponds to the inventory holding cost [52]

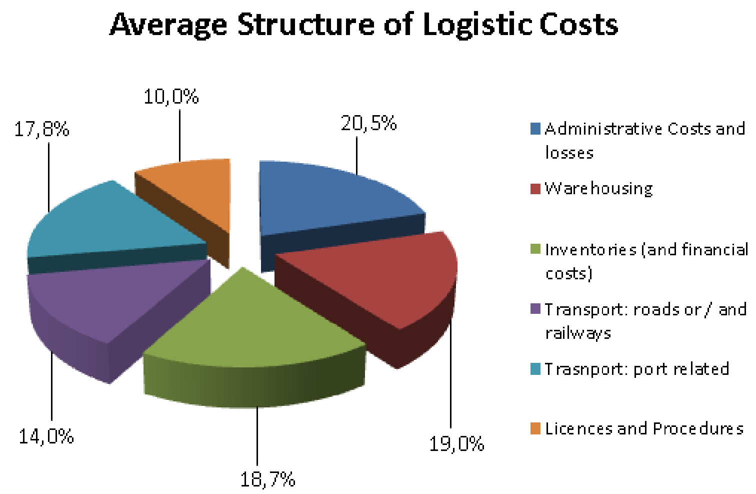


Figure 4: Structure of logistics costs [51]

according to Timme and Williams-Timme [118] quoted by Moin and Salhi [88]. In addition, it is important to note that the inventory control has to balance conflicting objectives due to two main reasons: i) economies of scale and purchasing batch and ii) uncertainty in offer and demand with production and transport lead time inevitably create the need for safety stock.

Table 2: Impact generated by the reduction of logistics costs [51]

Sector	Demand Increases	Employment Increases
Agro-Industry	9%	5%
Wood and Furniture	10%	12%
Textiles	6%	7%
Leather and Shoes	12%	10%
Mining	7%	2%

In the specific case of Spain, Globalog [49] found opportunities for improvement in the following aspects: i) inventory practices such as coding, classification, management of multi-echelon and Just In Time policy among others, ii) inventory management as modularity, backorder, Vendor Management Inventory (VMI) and demand planning and iii) practical network optimization and warehouse management as cross-docking and traceability among others.

A supply chain (SC) is defined as the system of organizations and flows of products, information and money spanning over the product's entire life cycle, from conception to consumption and final disposal. It involves different actors such as suppliers, producers, distributors, transporters and retailers, among others. All of them involved directly or indirectly in satisfying the request of a final client. Coordination and integration of SC activities are now recognized as critical to obtain competitive advantage [88]. Guasch and Kogan [52] observe that the logistics performance directly affect the cost of the products and hence the overall performance of an industry. These authors compare two economic organizations in order to find the impact of logistics cost on the price of a product, and conclude that the costs associated with inventory management represent about 19% of the price of a product in countries with poor logistics systems, compared to 8% in countries with efficient logistics networks. Guasch [51] conclude that the reduction in logistics costs can be evidenced by two indicators, namely 1) the increase in the proportion of demand of a product, and 2) the increase in the proportion of employment that can be generated for some sectors of the economy. As a consequence, the increase in product price obstructs the competitiveness and complicates the maintenance of inventory.

Logistics costs can be categorized administrative, warehousing, inventory, transportation and licenses costs. Guasch [51] identifies that more than 69% of these are directly related to transportation and inventory. Inventory management alone represents a proportion of net operating assets of approximately 37% in industry, 62% in distribution and 56% in retail [118]. In addition, it is important to note that the inventory control has to balance conflicting objectives due to two main reasons: 1) economies of scale and purchasing large batch sizes, and 2) uncertainty in supply and demand, which inevitably create safety stocks.

2.2 Information management in the supply chain

The management and coordination of the information between the processes in the SC is really important in order to take decisions. In Gavirneni, Kapuscinski, and Tayur [47] the flow of information between a supplier and a client is analyzed, for this, three situations are considered: i) there is no information for the supplier before the request comes, ii) the supplier knows the policies that the client uses as well as the final distribution processes and iii) the supplier has all the information about the state of the client. The costs analysis indicates that the second configuration reduces 50% of the cost compared to the first configuration. When the second and the third are compared, the cost reductions change in a wide range between 1% and 35%.

According to Psaraftis [93], the four dimensions of the information are evolution, quality, availability and processing. Just the first and second components add randomness. The evolution that the information experiences over the time highlights that this can change during the execution of the preliminary planning, and its quality reflects the possibility of the existence of some amount of uncertainty and asymmetric information between actors or entities.

The demand information experiences variability and amplification along the SC. This effect is known as the Bull Whip Effect (BWE). Giard and Sali [48] argued that the BWE is the main reason of the efficiency and efficacy loose in the SC. Chopra and Meindl [29] states that the BWE can be damped by an improvement on the operative performance and the design of rationing schemes in the products that present shortage. In this way, some of the proposals are: reducing the replenishment lead-time, reducing the lot size and taking into account historical data and the interchange of information to limit the variability.

Accurate and timely information management can optimize the performance of the SC. According with Wagner [121], although the SC involves several activities like purchase, production, localization, marketing, inventory control and distribution, the deep roots of the integration in the SC are in the last two activities, which are focused in the efficiency of the channel and they coordinate the performance of the individual entities in the satisfaction of the final client.

One way to reduce the effect of variability in the information is to assign the responsibility for management between activities to a single actor. This is achieved

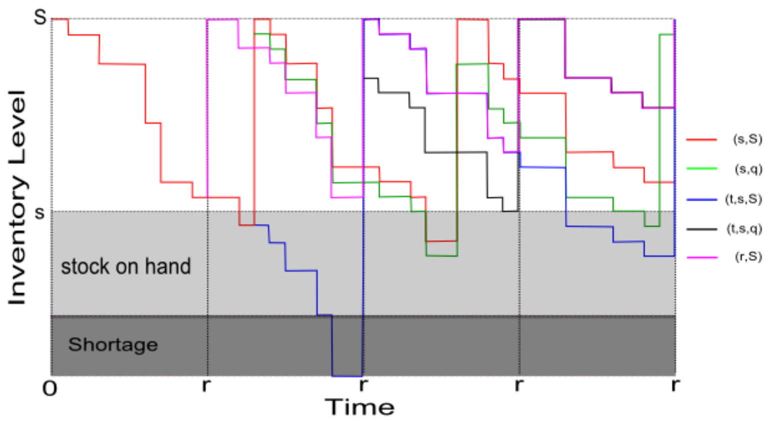


Figure 5: Inventory policies for ordering

with policies like Vendor Managed Inventory (VMI), which requires that the information between actors be shared, in special with the actor who is going to take the decisions.

2.3 INVENTORY POLICIES

Policies use to be based on three parameters, that can be related to the key questions to solve inventory control: when replenish, how much to replenish and how often the inventory level is reviewed. Wensing [124] highlights five policies according with these parameters, which are described below. Figure 5 shows the behavior the each policy in the time.

1. The policy (s, S) consists in ordering a variable quantity equal to the difference between a value S and the current inventory position as soon as the inventory level is less than a value s.
2. In (s, q), a fixed quantity q is ordered as soon as the inventory level is less than a value s.
3. In the policy (t, s, S), the inventory level is revised in each time period t. In case the inventory level is less than s, a quantity is ordered. The quantity ordered Q is established from the difference between a fixed value S and the current inventory level.
4. In (t, s, nq) each time period t the inventory level is revised, a multiple n of the fixed quantity q is ordered if the inventory level is less than a value s.
5. In the policy (r, S), the inventory level is reviewed each time period r and the difference between a fixed value S and the current inventory level is ordered.

2.4 STOCHASTIC DEMAND AND LEAD TIME MODELING

Models of demands with Poisson were presented by Axsäter [10] and Axsäter [11]. Its objective is to evaluate the total system costs for different inventory policies, and to optimize the system. The result is an accurate methodology for analyzing inventory costs. In Chao and Zhou [22], it is addressed an inventory system with continuous review in infinite horizon, where the sales price and inventory replenishment are determined simultaneously. The demand process is modeled by a Poisson probability distribution, with a arrival rate that depends on the price.

In the case of Normal probability distribution, in Berling and Marklund [14] an approximate model for coordinating inventory control of a warehouse and multiple clients is addressed. Results show a reduction in the holding inventory cost at least 30% the analyzed case study.

Queueing models have been used for representing systems in a SC. Saffari, Asmussen, and Haji [103] considered a M/M/1 queuing system with a (s, q) inventory policy and the possibility of lost sales, in which the demands arrive according to a Poisson's distribution and service times modeled by an exponential probability distribution. The aim of the study is to obtain reorder points and optimal quantities order for several cases. They found that there is no difference between the length of the queue size in the steady state model and in the classic M/M/1. Schwarz et al. [108] addressed the study of M/M/1 queues with attached inventory. The study considers Poisson-distributed demand and service/lead times exponentially distributed.

In Saffari and Haji [104], it is presented a model for a SC consisting of two levels for one supplier and several clients. They get the measures of long-term performance of the system and present an optimization model to determine the parameters for an inventory policy (s, q).

Another model for an inventory system with two suppliers is proposed by Song and Zipkin [113], where one supplier responds best to demand that the other. One of the nodes has a limit on its occupation, so, when an unit that arrives exceeds its limit; it just bypasses the node. For the two suppliers is assumed constant lead time. The demand is modeled as a continuous time flow with Poisson probability distribution and linear ordering costs.

The impact of lead time on the inventory cost is analyzed in a model for single product in continuous time by Song [115]. The variables of interest are the inventory level and the behavior of average long-term cost. The study shows that a short lead time requires higher inventory level. However, a higher lead time will not necessarily result in a higher average cost.

2.5 OPTIMIZATION METHODS FOR THE INVENTORY AND DISTRIBUTION

Optimization methods require information of current and historical inventory levels, the behavior of the demand, the location and the transport costs, as well as the capacity and availability of vehicles and drivers for deliveries. With this information it is possible to find optimal solutions or, at least, near optimal solutions of distribution and transport cost.

Coordination between inventory and transport can be faced from two perspectives: i) from the transport process, where inventories constraints are added to Vehicle Routing Problems (VRP) argued in Labadie and Prins [73], or ii) to approach it as a variant of a problem of control of production and inventory, where a vehicle plays the role of the production system in accordance with the arguments presented in Reiman, Rubio, and Wein [97]. With both perspectives, the importance besides in the interest of calculate the marginal profit (revenue minus delivery cost) for each client and the delivery cost (routes, clients selection and the quantities allocated for each client).

The IRP works as a component integration element and according to Kleywegt, Nori, and Savelsbergh [70], the IRP is one of the fundamental problems to be solved in the application of business practices as VMI. An overview of IRP is provided by Bertazzi, Savelsbergh, and Speranza [17] and Moin and Salhi [88].

Using the first approach and in terms of complexity, it is possible to decompose the IRP. This may be originated from the needed of selecting the best route along the clients. This problem is known as Travelling Salesman Problem (TSP), a classical combinatorial optimization problem, whose details may be consulted in Matai, Mittal, and Singh [85]. It is also necessary to add restrictions to routes to be followed by vehicles, then this problem becomes in a VRP. The VRP, its variants and features can be consulted in Toth and Vigo [119]. Thus, when the levels of consumption of the clients and the need of suppliers to maintain a continuous replacement are considered in the model, an IRP system is created. An IRP fulfills three aims: i) To establish the optimal inventory levels. ii) To plan volume and number of shipments and iii) to ensure that deliveries to suit the requirements of each product.

In the search for the solution of the IRP, it is possible to seek the exact solution, which usually takes a considerable investment of time calculation. The other option is to search for a feasible solution (not necessarily the best) in a reasonable time, although is possible to find in the literature hybrid approaches.

Heuristics and metaheuristics have been used in the search for a feasible solutions in complex spaces. Evolutionary algorithms are widely used. These meta-heuristics can be differentiated into a quite large set of algorithmic families representing bio-inspired system, which mimic natural evolution. Simic and Simic [109] argued that the complex optimization problems as IRP can be solved successfully by hybrid approaches with techniques such as artificial neural networks, genetic algorithms, tabu search, simulated annealing and evolutionary algorithms. Some bio-inspired techniques to solve IRP are summarized in the Table 3.

An alternative to face complex problems is to decompose them. Thus, Archetti, Doerner, and Tricoire [7] proposes a model that includes inventory control, routing and delivery scheduling. The model is solved by decomposing the problem into two phases, the first one is to create a scheduling and the second one is the design of the routes. The second phase uses a VRP model with time windows. Variable Neighborhood Search is used which is implemented by two destruction operators, one to remove selected randomly travels and the other to eliminate stations.

In Christiansen et al. [30] the method is conformed by two components: a heuristic construction algorithm and a genetic algorithm. The construction algorithm builds a plan from scratch. It is deterministic, but has parameters that can be varied to produce different plans. The genetic algorithm is used to search for parameters that produce good plans by the construction heuristic.

Other application of genetic algorithms is the clustering of retails into m groups in accordance with the number of vehicles available. In the Cheng and Wang [26] this clustering information is then passed to sub-problems, and each sub-problem optimizes its own routing sequence for replenish retails.

Metaheuristics such as simulated annealing and local search are used to evaluate and improve initial solutions. In Li et al. [76], Liu and Lin [80] and Sajjadi and Cheraghi [105] Simulated Annealing is used to improve initial solution obtained from other heuristics and metaheuristics. In Qin et al. [94] local search methods are used for to insert and to removal new replenishment points into a retailer's schedule.

Table 3: Metaheuristics used for IRP

Technique	Use	Ref.	Use
Artificial	Demand forecasting.	[67]	To predict the behavior
Neural	Price forecasting.	[91]	of a variable interest
Networks	To classify units of inventory.		
	To search for good parameters		
	for a function or heuristic.	[30]	
Genetic	Clustering retails to replanish by	[26]	To find good solutions
Algorithms	each of the vehicles available.	[79]	in large search spaces
	To search optimal routes for		
	replanishment retails.		
	Replenishment policy for inserts		
	and removal new replenishment		
	point into a retailer?s schedule.	[94]	To avoid search of
Local	Adjust the quantity to delivery to	[68]	solutions in spaces
Search	retailer's.		previously visited
	Exclusive operators for solve		
	special cases.		
Simulated	To improve an initial solution	[76]	To avoid premature
Annealing	obtained from other heuristics	[80]	solutions which are
	and metaheuristics.	[105]	not good enough

Li et al. [76] focused the study on minimizing travel times in a context related to law regulations about hours of service. The problem addressed considers a vendor, multi-client, homogeneous fleet of vehicles and estimation of deterministic demand. To evaluate the performance of the system, a Lagrangian relaxation approach was used in order to obtain a lower bound for the solution of the problem. Compared to this, the tabu search algorithm used shows to be close to the lower limits of the problems for small to medium size.

The model presented in Agra et al. [3] includes multiple products, multiple suppliers, multiple clients and heterogeneous fleet capacity. Also multiple period estimation and deterministic demand. All this was used three heuristic methods: rolling horizon (RH), local branching (LB) and feasibility pump (FP). In the case of RH the planning horizon was decomposed into smaller time horizons. In the other hand, LB seeks for local optimal solutions by restricting the number of variables that can change their values and FP seeks for initial feasible solution. The results show better solutions than those obtained with a single heuristic.

When IRP deals with stochastic demand, Bertazzi et al. [15] proposes a model consisting on one supplier and a set of clients. A hybrid algorithm is used to solve the problem. The estimated cost are obtained joining the exact solution of a mixed integer linear programming with the branch and cut heuristic.

Another important variability term faced by an IRP model is the variability in the travel times, which requires extra work on non-deterministic and probabilistic approaches for some instances. In Reiman, Rubio, and Wein [97], the travel time between clients is represented in the random variable by the idle time.

Dealing with complex problems, such as the IRP, it is common to place a set of instances or testing problems available for other. Following this approach, Papageorgiou et al. [90] creates a library composed of test instances for the maritime IRP, it also creates a virtual community for discussion on topics such as mathematical models of linear mixed integer programming, providing so, a framework with common characteristics for this type of problems. Resources are available online[1]. In the specify case of routing vehicles, it is also possible to find the instance set proposed by researchers belonging to CIRRELT, SCL, OR@Brescia and Logistics Management Department of Helmut-Schmidt-Universitat, online [2] [3] [4] [5]. Other resources can be finding in ths site by Adulyasak online [6]. Instances were created for variants of the problem of inventory and routing such as stochastic, dynamic, multi-product and multi-vehicle along others.

New tends for Intelligent Data Analysis are presented in Corchado et al. [44].In the first contribution applies principal component analysis for quantitative association rules' quality. From this analysis, a reduced subset of measures is selected to be included in the fitness function in order to obtain better values for the whole set of quality measures, and not only for those included in the fitness function. Other contributions are technical for bio-inspired knowledge system for calculating parameters of functions.

According to Bertazzi et al. [15] the trend in IRP is to study it as a model system, contrary to what is traditionally works IRP decomposition into simpler problems.

1 http://mirplib.scl.gatech.edu/
2 http://www.leandro-coelho.com/instances/
3 http://www.tli.gatech.edu/research/casestudies/irp2/
4 https://sites.google.com/site/orbrescia/home
5 http://www.hsu-hh.de/logistik/
6 https://sites.google.com /site/yossiriadulyasak/

The working direction followed in IRP is to analyze the problem of distribution and inventory control as a whole system. This is due to advances in the methods of the solution both heuristic and exact and the increasing power of commercial software for linear programming and mixed integer linear programming.

2.6 conclusions

Inventory policies are the core of the problems of inventory routing problems since they determine the modeling of the problem but the objective function used, the restrictions set as well as the optimization techniques used, play also an important role. They determine the level of service of the SC, where the key is to correctly study the behavior of the demand and the lead time in order to achieve a useful representation of the system to take proper decisions.

The IRP is in the middle, between strategic, tactical and operational decisions of the SC. Strategic, because it supports the policies governing the management, indicators and targets related to business needs. Tactical, because it generates procedures to be followed, monthly goals and work plans. Operational because it is required to feedback to the system in order to keep it under control with the dynamic adjustments the actual work load needs.

3 SCIENTOMETRIC ANALYSIS ON DSIRP

The popularity and rapid development of Inventory Routing Problem as an area of research has led to a huge amount of publications containing the achieved knowledge. Due to the interdisciplinary nature of inventory routing problems that include heuristics, operation research and management science, a scientometric study of the area should shed some light on the topic. Empirically studying the evolution of the research on this field will give a view about groups of researchers, their research productivity and impact, all that providing a better understanding of trends. While scientometric has a long tradition in many fields, we identify a lack of comprehensive studies in the area of inventory routing. Based on bibliographic databases (Scopus and Web of Science), this study applies a scientometric method to empirically analyse the evolution and state of the Inventory Routing Problem research. We focus on analysing variants of the problems where the inventory is revised periodically and the decision making is affected by the dynamic variation of the demand that is revealed as time goes by in the planning horizon. The results of this study provide a better understanding of patterns, trends and other important factors as a basis for directing research activities, sharing knowledge and collaborating in the operations research area. It also makes reference to the transversal areas such as the mathematics and computer science.

3.1 INTRODUCTION

The inventory administration, control and distribution is a topic of interest for the researchers for the last three decades [37]). A great number of papers are written every year proposing operational and tactical decisions in the supply chain, where the coordination of inventory and transportation activities is important to gain competitive advantages [88]. This problem is known as inventory routing problem, a classical optimization problem that puts together different knowledge areas.
By considering the number of publications in the last ten years, we observe a continuous and incremental interest in this topic, it becomes more and more important to investigate the current state and evolution of IRP. One manner to analyze this evolution is by means of the Scientometric study. According to Lewis, Templeton, and Luo [74], the Scientometric allows to quantize the studies as well as measuring and analyzing science activities. Also, the Scientometric studies facilitate the development and improvement of an academic discipline, serving as a vital basis for defining and debating future research agendas [54].
The identification of research areas has been a perennial theme in scientometric. A research area is defined as a set of papers or other biometrics units that define a research topic and associated groups of research who share an interest in the topic [110]. Using an scientometric study, it is possible to find information about research activities in general, such as knowledge sharing, research quality, socio-organizational structures, influential countries, affiliations, authors, development of key topics, structural change, and economic impact of research that guide its work.
Due to the complexity of IRP, its analyzing require two elements that need to be

taken into account. The first one consists of the versions of the problem that are involved in the study and the second involves the knowledge areas these versions are searching. The interest in this article is to analyze the versions of the problem that have Dynamic and Stochastic demand with revision of the inventory in period form. The study is focused on the subject areas related to operational research, social sciences and business science.
In the literature, a lack of scientometric studies in IRP and its stochastic versions has been detected. However, there are review papers which will be taken as a base to find the key factors for the analysis. Those articles contain a large number of references which are related and will be classified and analyzed. But still so, these reviews lack the elements that distinguishes scientometrics studies such as: analysis of publications per category, analysis of citations per category citation, identification of the authors and the most representative groups of researchers among others.

As a stating point we have identified 8 of the most relevant review papers in our opinion, according the amount of citations that these received. Also, we have highlighted the amount of papers that used in its studies and the specific key factors for the analysis that these used, which will be discussed below. We our study besides review the papers in IRP allows to know the state of the art of research of IRP, especially in stochastic, dynamic inventory routing problems with the periodic revision of the inventory as well as its trends and patterns.

In Melo, Nickel, and Saldanha-Da-Gama [86] the decision on factory locations involved in IRP are considered. They classified 139 papers by means which identified four basic features that may be included in a facility location model to make it useful in strategic supply chain planning: multi-layer facilities, multiple commodities, single/multiple period(s), deterministic/stochastic parameters. Also, they classified the literature according to some typical supply chain decisions namely, capacity, inventory, procurement, production, routing, and the choice of transportation modes. Though their classification clearly show that facility location is frequently combined with inventory and production decisions and less frequently with procurement, routing and the choice of transportation modes.

Li and Wang [77], Andersson et al. [4] and Moin and Salhi [88] each cited 63, 125 and 49 papers respectively, Those works were emphasizing coordination mechanisms in the supply chain and inventory management and distribution. In Christiansen et al. [32] and Song and Furman [112], a review of trends in Maritime IRP is analyzed with 132 and 12 articles cited respectively. In Ko, Tiwari, and Mehnen [72] some applications of the soft computing in the IRP are analyzed with 188 papers cited. Ignaciuk and Bartoszewicz [63] analyzes strategics of efficient supply for logistics systems in the industry of perishable food, In this study, a dynamic and stochastic system was considered with the use of 35 references. In contrast, in our work, we use more than 1000 bibliographic registers and cite more than 50 highlighting papers for the scientometric analysis.

The main objective of this study is to provide a more comprehensive view on the Inventory routing area within a relevant time frame of the last ten years in order to present empirical and relevant findings. We focus on analyzing the IRP where the inventory is revised periodically and the decision making is affected by the dynamic variation in the demand. Information about the demand is revealed as time goes by in the time horizon of planning. Therefore, in this paper we present a comprehensive scientometric study that empirically explores publications related to Inventory Routing Problem covered by Elsevier's Scopus and Thompson's Web of Science databases from 2005 to 2014. We will analyze 934 papers cited by Scopus and 720 papers cited by Web of Science.

The remainder of the paper is organized as follows: Section 3.2 focuses on the methodology used for processing data. Section 3.3 is devoted to analyze publications patterns. Citation patterns are reviewed in Section 3.4. Groups of investigators are identified in Section 3.5. The paper is ended with conclusions, which emphasizes the current status, trends and patterns found for this field of study.

3.2 COLLECTION OF RELEVANT PUBLICATIONS ON DSIRP

The collection of relevant publications and citations establishes the foundation for a scientometric analysis of a specific research area. As indicated before, this study intends to cover a large part of peer-reviewed Inventory Routing Problem articles published in the last ten years, specifically, our focus is on documents related to the dynamic case for a periodically revised inventory. By this, we aim to obtain empirical evidence for supporting the metascientific findings of this scientometric study. In this section we describe our procedure regarding data collection and data processing and knowledge extraction.

3.2.1 *Data Collection*

The data was recollected by means of Elsevier's Scopus and Thomson's Reuters Web of Science databases. These databases were chosen for their relevance and reputation in the fields of physical and social sciences. We analyze each database separately to thereby complement and validate the results obtained.

A search equation was created in order to cover a large part of publications in Inventory Routing Problem and in the specific topics that we want to deepen. The equation was conformed by meanns of keywords chosen for their relevance regarding three key questions for the problem: a) how the inventory is controlled, b) how often the inventory is revised and c) how the information about the demand is reveled. For a two words INVENTORY and ROUTING were selected ; for b another two words POLICIES and PERIOD were selected, and for c only STOCHASTIC was selected. The equation was used for search query in the title, abstract and keywords for the documents contained in both bibliographic databases mentioned above.

The logical structure of this equation was (Inventory AND (Routing OR (Periodic AND (Policy OR Stochastic)))), thus, three sets of words were considered: a) Inventory – Routing, b) Inventory – periodic – policy and c) inventory – periodic – stochastic. The union of the data collected by these sets conformed the data base under study. The basic idea behind this specific equation is to get the documents related all the knowledge areas involved in the subject under study. The search equation was applied on several specific subject areas. Seven of them were chosen for each database due to their relevance in the field under study. Three of them were common for both databases: mathematics, computer science and engineering, but the others vary according to the categorization that each database had. In general, subject areas related to operational research, social sciences and business science were included.

For the period of time from the year 2005 to 2014, the search query found 934 publications by Elsevier's Scopus and 720 by Thomson-Reuters's Web of Science (see Table 4). A trend line of number publications per database and year can be observed

Table 4: Number of publications per database and year

Data Source	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Total
Scopus	50	50	59	99	95	116	95	101	125	144	934
WoS	44	54	45	70	76	70	67	84	98	112	720

in Figure 6, where we can speak of a growing number of publications and increasing interest by the researchers. It is important to note that although the information obtained have some similar registers, the work with each database independently, instead of be redundant was complementary.

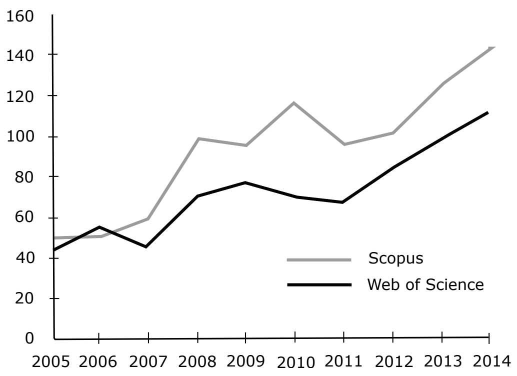


Figure 6: Trend line of number publications per database and year

3.2.2 *Data Processing and Knowledge Extraction*

We used several attributes to analyze frequency, productivity, quality and impact of the research into of the extensive information registers that were obtained. Below, we described briefly, the methods that we have used in the data processing to obtain trends and patterns, to highlight documents and researchers groups in specific topic among other relevant information.

The data processing begins with the form in which the registries of the data base are counted according to attributes that are analyzed. In most of the cases a simply counting was sufficient, for these cases, a measure of frequency that coincide with the criteria under related to was obtained and denoted with `f`. Also, for many other attributes it was worthy to calculate the relative frequency denoted with `fr`. These metrics could be used for the complete time horizon , namely ten years, or could be presented per year.

For measuring the productivity and according with Heilig and Voss [54] in their review of the literature, two could be of the methods to measure research productivity for the authors: straight count of the number of publications denoted by SCM and author position in the author list, denoted by APM. The SCM as its name indicates, assigns a score equal for each authors who his registered in the publication. APM instead, assigns a higher score to the first author and the score decreases as the author reaches the last position. We have used only these methods, because consider that they both preserve properly the concepts we want to analyze.

The impact that one publication generates in the research community is a measure of its quality. We measure this impact counting the number of citations received by the publication jointly with the longevity of the publication. Thus, we used the metric NCII not only for this purpose but also to analyze the quality of the research through the impact on new publications. We have used a NCII weighted denoted NCIIW. NCIIW consists of using the metric that counts the number of citations that one author has received by each publication depending on the author position, namely metric APM, and then, joining it by the metric NCII.

Others metrics used were: Number of citation in Google Scholar (GS); SJR a measure of scientific influence of scholarly journals that accounts for both the number of citations received by a journal and the importance or prestige of the journals where such citations come from; SNIP measures contextual citation impact by weighting citations based on the total number of citations in a subject field; the impact factor IF and the impact factor of the last 5 years 5YIF.

To ensure the accuracy of the results, the generated outputs are validated by manual proof-reading activities, this way inconsistencies can be identified. This semi-automatic process guarantees the quality of the results of the study.

3.3 ANALYSIS OF THE NUMBER OF PUBLICATIONS PER CATEGORY

In order to obtain patterns in the subject under study, the fields of information databases by the number of publications were analyzed. For this purpose we used specific perspectives such as academic disciplines, authors distributions, forms of publication and publications by authors.

3.3.1 *Academic Disciplines*

The general structure and development of the Inventory Routing Problem research can be observed through of the academic disciplines involved (see Table 5 and Table 6). At first, the average of publications percentage for a time horizon of ten years, we can state that in the part five of average of number of publications by year can be classified into at least one of the following three academic disciplines: Decision science, Engineering and Operation research and management science. Secondly, the first transversal areas that contributed in the research in area under study are the Computer Science and Mathematics, this demonstrates their importance of the problem formulation and the techniques of solution. Finally, can be seen that for no more than 6% of the publications was incorporating more specific areas such as: Social science, Automation, Transportation, Economic, Econometric and finance.

3.3.2 *Forms of the publication*

The articles were the most common type of document analyzed in both databases. In average, the were about 61% of documents per year in the Scopus database, whereas in Web of Science they were about 75%. This difference for the number of articles is compensated with the number of documents that are a result of the participation in conferences like articles, conference review and proceedings with about 30% and 20% respectively. These result can be observed in the Tables 7 and 8.

Table 5: Percentage of documents that were generated for each subject area per year on Scopus database

Subject Area	2005 %	2006 %	2007 %	2008 %	2009 %	2010 %	2011 %	2012 %	2013 %	2014 %	Av %
Decisions Sciences	29.76	26.44	21.10	26.80	25.60	21.82	24.63	23.70	26.27	22.58	24.87
Engineering	17.86	32.18	27.52	19.59	19.81	24.55	20.20	24.17	23.92	26.45	23.62
Computer Science	8.33	5.75	8.26	18.04	23.67	20.45	19.21	21.33	15.29	15.81	15.61
Mathematics	14.29	12.64	15.60	10.82	11.59	15.00	12.81	15.64	16.08	10.97	13,54
Business, management and Accounting	9.52	9.20	11.01	10.31	13.04	10.45	16.26	11.37	12.94	16.13	12.02
Scocial sciences	9.52	9.20	14.68	9.79	2.42	3.18	1.97	2.37	1.18	3.55	5,79
Economics, Econometric and Finance	10.71	4.60	1.83	4.64	3.86	4.55	4.93	1.42	4.31	4.52	4.54

Table 6: Percentage of documents that were generated for each subject area per year on Web of Science database

Subject Area	2005 %	2006 %	2007 %	2008 %	2009 %	2010 %	2011 %	2012 %	2013 %	2014 %	Av %
Operations Research & Management Science	36.14	37.50	47.62	43.57	38.26	44.17	42.40	40.00	43.86	42.36	41.59
Engineering	16.87	20.19	15.48	22.86	20.13	15.83	25.60	18.67	25.15	22.66	20.34
Business & Economics	20.48	22.12	22.62	20.71	16.78	19.17	12.00	17.33	14.04	16.75	18.20
Computer Science	15.66	14.42	5.95	7.86	16.78	10.83	12.80	14.67	9.36	8.87	11.72
Mathematics	6.02	2.88	3.57	1.43	3.36	5.00	2.40	2.67	1.17	3.45	3.19
Automation & Control Systems	3.61	2.88	1.19	2.14	4.03	3.33	1.60	4.67	4.09	1.97	2.95
Transportation	1.20	0.00	3.57	1.43	0.67	1.67	3.20	2.00	2.34	3.94	2.00

Table 7: Percentage of documents classified by type in Scopus database

Document Type	2005 %	2006 %	2007 %	2008 %	2009 %	2010 %	2011 %	2012 %	2013 %	2014 %	Av %
Article	42.00	70.00	69.49	61.62	63.16	58.62	48.42	65.35	65.60	58.33	60.26
Conference Paper	36.00	24.00	27.12	34.34	29.47	34.48	37.89	16.83	25.60	19.44	28.52
Review	8.00	4.00	1.69	1.01	2.11	3.45	8.42	14.85	4.00	3.47	5.10
Conference Review	4.00	0.00	0.00	3.03	5.26	2.59	1.05	2.97	3.20	5.56	2.77
Article in Press	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.60	13.19	1.48
Book Chapter	6.00	2.00	0.00	0.00	0.00	0.86	2.11	0.00	0.00	0.00	1.10
Book	2.00	0.00	0.00	0.00	0.00	0.00	2.11	0.00	0.00	0.00	0.41
Short Survey	0.00	0.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17
Note	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20

Table 8: Percentage of documents classified by type in Web of Science database

Document Type	2005 %	2006 %	2007 %	2008 %	2009 %	2010 %	2011 %	2012 %	2013 %	2014 %	Av %
Article	54.55	57.41	88.89	61.43	76.32	88.57	68.66	89.29	85.71	85.71	75.65
Proceedings Paper	29.55	22.22	8.89	25.71	19.74	7.14	16.42	4.76	4.08	5.36	14.39
Article; Proceedings Paper	13.64	20.37	0.00	10.00	1.32	0.00	11.94	1.19	5.10	4.46	6.80
Review	2.27	0.00	2.22	2.86	2.63	4.29	2.99	4.76	4.08	3.57	2.97
Editorial Material	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02	0.89	0.19

Table 9 shows a list of journals and the related number of publications. Only taking into account the European Journal of Operational Research and International Journal of production economics, the sum of their participation in the two databases is about 15% in Scopus and 23% in Web of Science. IRP being an optimization problem, there are journals of transversal areas to operational research area as is the computer science. This is in particular evident in the database Web of Science with journals as Computer & Operations research and Computers & Industrial Engineering. Also, this is showed, in Table 10 where the five most important conferences appear. Besides finding specialized conferences for industrial engineering, it appears also a conference related the industrial engineering with the computer science area (International Conference on Computers and Industrial Engineering) that shows the relevance of computers science area for IRP.

Table 9: Ranking of publications per journal

Scopus Database				Web of Science Database			
Rank	**Journal**	**f**	**fr%**	**Rank**	**Journal**	**f**	**fr%**
1	European Journal of Operational Research	73	7.79	1	European Journal of Operational Research	89	12.36
2	International Journal of Production Economics	64	6.83	2	International Journal of Production Economics	78	10.83
3	Journal of Optimization Theory and Applications	37	3.95	3	International Journal of Production Research	39	5.42
4	Flexible Services and Manufacturing Journal	30	3.20	4	Operations Research	34	4.72
5	Discrete Dynamics in Nature and Society	28	2.99	5	Computers & Operations Research	30	4.17
6	Socio-Economic Planning Sciences	20	2.13	6	Computers & Industrial Engineering	24	3.33
7	Transportation Science	17	1.81	7	Naval Research Logistics	20	2.78
7	Operations Research/ Computer Science Interfaces Series	17	1.81	8	Journal of the Operational Research Society	16	2.22

Table 10: Ranking of publications per conferences

	Scopus		WoS	
Conference Name	f	fr(%)	f	fr(%)
Annual Conference and Expo of the Institute of Industrial Engineers	21	8.24	–	–
International Symposium on Inventories	–	–	19	13.38
International Conference on Computers and Industrial Engineering	17	6.67	–	–
International Conference on Industrial Engineering and Engineering Management	12	4.71	16	11.27
Winter Simulation Conference	10	3.92	–	–

3.3.3 *Publications by Authors*

With the aim to emphasize the authors with the highest number of publications and the position that every of these authors has in the publication list of authors, Table 11 and Table 12 were created. In these tables, the number of publications per each author and their position is considered as first, second, third, fourth or fifth coauthor. Afterwards, we use two different metrics to show their production: i) the SCM, assigned the same punctuation to all publications reported, in this case of 1, by each author are added and the results are shown in a column with the same name. ii) this second method applies APM in which a different punctuation is given for every position, being the first position the one with the highest punctuation and successively descending this punctuation for the next positions until every punctuation is calculated for every author. Taking into account that the highest number of authors gathered article in the sample is 8, the maximum punctuation is this number. For each author, their punctuations are multiplied by the number of publications respectively and then the total obtained. The results are in the corresponding column for both tables ordered in decreasing order of total punctuation by author.

Ignaciuk przemystaw, Bartoszewicz Andrzej and Laporte Gilbert are the most prominent researchers. The first two researchers linked the research areas of computer science and operations research, this is evident in their research topics for the first one are logistic systems and supply chain management, congestion control in data transmission networks, networked control systems, dynamical optimization and robust control and for the second one are sliding mode control and congestion control in communication networks. Regarding Laporte Gilbert in his expertize as reported by HEC Montreal [1] are in Combinatorial optimization, Transportation and Operational research. Also, according with google scholar other areas off his interest are distribution management and mathematical programming.

Table 11: Top sixteen authors by APM metric in Scopus stating the number of publications in each of the positions of authorship and using SCM and APM metrics

Rank	Author	1st	2nd	3rd	4th	5th	SCM	*APM
1	Ignaciuk Przemystaw	15	–	–	–	–	15	120
2	Bartoszewicz Andrzej	–	16	–	–	–	16	112
3	Laporte Gilbert	–	8	8	–	1	17	108
4	Aghezzaf El-Houssaine	3	9	1	–	–	13	93
5	Chen Haoxun	1	5	8	–	–	14	91
6	Li Jianxiang	8	1	2	–	–	11	83
6	Christiansen Marielle	3	4	3	1	1	12	79
7	Chu Feng	–	7	4	1	–	12	78
8	Zhou Sean X.	4	5	1	–	–	10	73
9	Chao Xiuli	5	3	2	–	–	10	73
10	Coelho Leandro C.	9	–	–	–	–	9	72
11	Cordeau Jean-Francois	1	8	–	1	–	10	69
12	Bertazzi Luca	5	3	1	–	–	9	67
13	Savelsbergh Martin W. P.	2	2	3	3	–	10	63
13	Zhang Jiang	5	2	1	–	–	8	60
13	Wang Li	4	4	–	–	–	8	60
14	Louly Mohamed-Aly Ould	5	2	–	1	–	8	59
15	Cannella Salvatore	5	2	–	–	–	7	54
16	Song Jin-Hwa	2	3	1	1	1	8	52
16	Chen Yuerong	3	4	–	–	–	7	52

*the data with the same APM score, also obtained the same rank position

3.4 ANALYSIS OF CITATIONS PER CATEGORY

Bearing in mind that the number of citations shows how often the publication is referenced by other publications, in this sections we will be analyzing the impact

1 http://www.hec.ca/en/profs/gilbert.laporte.html

Table 12: Top sixteen authors by APM metric in Web of Science stating the number of publications in each of the positions of authorship and using SCM and APM metrics

Rank	Author	1st	2nd	3rd	4th	5th	SCM	*APM
1	Laporte Gilbert	–	7	8	–	1	16	101
2	Zhou Sean X.	5	5	1	–	–	11	81
3	Chao Xiuli	4	3	4	–	–	11	77
4	Coelho Leandro C.	9	–	–	–	–	9	72
5	Bertazzi Luca	5	3	1	–	–	9	67
6	Aghezzaf El-Houssaine	3	5	1	–	–	9	65
6	Christiansen Marielle	2	4	2	1	1	10	65
7	Savelsbergh Martin W. P.	2	1	3	3	–	9	56
8	Cordeau Jean-Francois	–	7	–	1	–	8	54
9	Song Jin-Hwa	2	3	1	1	1	8	52
10	Louly Mohamed-Aly Ould	4	2	–	1	–	7	51
11	Chen Youhua (Frank)	2	4	1	–	–	7	50
12	Archetti Claudia	6	–	–	–	–	6	48
13	Speranza Maria Grazia	–	2	2	4	–	8	46
13	Huh Woonghee Tim	4	2	–	–	–	6	46
13	Chew Ek Peng	4	2	–	–	–	6	46
14	Chu Feng	–	4	2	1	–	7	45
15	Kiesmueller Gudrun P.	2	4	–	–	–	6	44
16	Janakiraman Ganesh	–	5	1	–	–	6	41
16	Cardos Manuel	2	1	3	–	–	6	41

*the data with the same APM score, also obtained the same rank position

of the citations and its patterns. For that, the number of citations received were analyzed for the time frame. The total number of citations obtained for the sample was 5724 in scopus and 4996 in web of science with a mean of publication citations of 6.11 and 6.94 respectively.

3.4.1 *Citations by document type*

The reviews most cited were Melo, Nickel, and Saldanha-Da-Gama [86] and Li and Wang [77]. In the first one, a complete review of strategics in the design of supply chain networks focuses in facility location was presented. Also, reviewed the optimization methods for solving facility location problems in a supply chain context and included practical applications of location models in SCM. And the second one, a review of coordination mechanisms of supply chain systems in a framework that is based on supply chain decision structure and nature of demand was presented. This framework highlighted the behavioral aspects and information need in the coordination of a supply chain.

The articles, specially the most cited ones, offer approximations to the boundaries of knowledge of every subject. For this reason, being the most published type of document and being among the most cited ones, we created a ranking of articles in descending order of metric Normalized Citation Impact Index (NCII) for each database in Table 14 and Table 15. The two articles most cited in Scopus were Mete and Zabinsky [87] and Coelho, Cordeau, and Laporte [37]. In the first one, it was develop a stochastic programming model to select the storage locations of medical supplies and required inventory levels for each type of medical supply. In the

Table 13: Top five review papers by NCII metric in both databases

		Scopus				Web of Science			
Year	Review	Rank	NCII	f	FG	Rank	NCII	f	FG
2009	Facility location and supply chain management: A review	–	–	–	–	1	44.67	268	752
2007	Coordination mechanisms of supply chain systems	–	–	–	–	2	16.38	131	332
2010	Industrial aspects and literature survey: Combined inventory management and routing	1	17.00	85	185	3	14.40	72	185
2013	Ship routing and scheduling in the new millennium	2	9.50	19	74	4	7.50	15	74
2010	A review of soft computing applications in supply chain management	–	–	–	–	5	5.80	29	70
2007	Inventory routing problems: A logistical overview	3	8.00	64	143	–	–	–	–
2013	A maritime inventory routing problem: Practical approach	4	6.50	13	47	–	–	–	–
2012	LQ optimal sliding-mode supply policy for periodic-review perishable inventory systems	5	5.00	15	20	–	–	–	–

second one, a comprehensive review of literature in IRP is presented. It is based on categorize IRP with respect to their structural variants and with respect to the availability of information on customer demand. Regarding to Web of Science, the reviews most cited were Chen and Vairaktarakis [25] and Yu and Egbelu [127]. In the first one, an integrated scheduling model of production and distribution operations by applications in the computer and food catering service industries were studied. The problem consisted in to find a joint schedule of production and distribution such that an objective function that takes into account both customer service level and total distribution cost is optimized. The second one aims to find the best truck docking or scheduling sequence for both inbound and outbound trucks to minimize total operation time when a temporary storage buffer to hold items temporarily is located at the shipping dock were found. Also, the product assignment to trucks and the docking sequences of the inbound and outbound trucks are all determined simultaneously.

3.4.2 *Citations by Journals*

With the aim of identifying the most specialized journals in IRP document publications, two tables (see Table 16 and 17) with the most cited journals was created for each database. The results show that the journals obtained appear with high impact factor usually in Q1 quartile for the ranking of publications classified by topics.

3.4.3 *The most cited authors*

The number of citations every author receives is important to detect those authors that most contribute to the growth of the specific subject area. To give an impartial indicator of the most important authors according to the number of citations received, the metric NCII is used again, adapting it as an individual productivity measure. Thus, we calculate the weighted metric NCII according as we have naming NCIIW. In this case the punctuation's authors are multiplied by the value of NCII instead that theirs number of publications.

Table 14: Top ten by NCII metric in Scopus database

Rank	Year	Article	*NCII	f	FG
1	2010	Stochastic optimization of medical supply location and distribution in disaster management	18.20	91	184
2	2014	Thirty years of inventory routing	12.00	12	73
3	2007	Incorporating inventory and routing costs in strategic location models	11.50	92	191
4	2005	Distribution network design: New problems and related models	11.10	111	238
5	2006	Vehicle routing scheduling for cross-docking in the supply chain	8.67	78	157
6	2010	Incorporating location, routing and inventory decisions in supply chain network design	8.40	42	91
7	2009	Inventory inaccuracy in retail stores due to theft: An analysis of the benefits of RFID	7.83	47	78
8	2012	The inventory-routing problem with transshipment	7.67	23	56
8	2012	On the Bullwhip Avoidance Phase: The Synchronised Supply Chain	7.67	23	33
9	2009	Vehicle routing with cross-docking	7.33	44	106
10	2010	LQ optimal sliding mode supply policy for periodic review inventory systems	7.20	36	45

*the data with the same NCII score, also obtained the same rank position

Table 15: Top ten by NCII of the best articles in Web of Science database

Rank	Year	Article	*NCII	f	FG
1	2005	Integrated scheduling of production and distribution operations	11.60	116	267
2	2008	Scheduling of inbound and outbound trucks in cross docking systems with temporary storage	10.00	70	200
3	2010	Stochastic optimization of medical supply location and distribution in disaster management	9.40	47	184
3	2010	Quality. safety and sustainability in food distribution: a review of quantitative operations management approaches and challenges	9.40	47	112
4	2014	Thirty Years of Inventory Routing	9.00	9	73
5	2005	Distribution network design: New problems and related models	8.60	86	238
6	2007	Incorporating inventory and routing costs in strategic location models	8.38	67	191
7	2013	The exact solution of several classes of inventory routing problems	6.50	13	44
7	2013	A maritime inventory routing problem: Practical approach	6.50	13	47
8	2007	Inventory routing problems: a logistical overview	6.25	50	143
9	2012	The inventory routing problem with transshipment	6.00	18	56
9	2009	Vehicle routing with cross docking	6.00	36	106
9	2012	Consistency in multi vehicle inventory routing	6.00	18	41
9	2007	A branch and cut algorithm for a vendor managed inventory routing problem	6.00	48	116
10	2010	A Branch and Price Method for a Liquefied Natural Gas Inventory Routing Problem	5.40	27	64

*the data with the same NCII score. also obtained the same rank position

Table 16: Journal citations in Scopus Database stating frequency f, number of paper n and relative frequency f/n, also include SJR, SNIP, IF and 5YIF metrics

Rank	Journal name	f	n	fr	SJR	SNIP	IF	5YIF
1	European Journal of Operational Research	1112	73	15.23	2.60	2.50	1.84	2.63
2	International Journal of Production Economics	677	64	10.58	2.39	3.20	2.08	2.59
3	Computers and Operations Research	373	28	13.32	2.97	3.03	1.72	2.34
4	Operations Research	371	37	10.03	3.45	1.93	1.50	2.50
5	Transportation Science	246	14	17.57	3.14	2.93	2.29	2.91
6	Journal of the Operational Research Society	188	11	17.09	1.39	1.23	0.91	1.27
7	Computers and Industrial Engineering	186	20	9.30	1.72	2.38	1.69	2.38
8	International Journal of Production Research	154	30	5.13	1.33	1.73	1.32	1.25
9	Manufacturing and Service Operations Management	131	13	10.08	2.64	1.58	1.45	2.69
10	Naval Research Logistics	105	17	6.18	1.13	0.74	1.04	1.24

Table 17: Journal citations in WoS Database stating frequency f, number of paper n and relative frequency f/n, also include SJR, SNIP, IF and 5YIF metrics

Rank	Journal name	f	n	fr	SJR	SNIP	IF	5YIF
1	European Journal of Operational Research	1356	89	15.24	2.60	2.50	1.84	2.63
2	International Journal of Production Economics	518	78	6.64	2.39	3.20	2.08	2.59
3	Computers and Operations Research	358	30	11.93	2.97	3.03	1.72	2.34
4	Operations Research	311	34	9.15	3.45	1.93	1.50	2.50
5	Management Science	199	8	24.88	3.65	3.10	1.73	3.30
6	Transportation Science	198	14	14.14	3.14	2.93	2.29	2.91
7	International Journal of Production Research	186	39	4.77	1.33	1.73	1.32	1.25
8	Journal of the Operational Research Society	176	16	11.00	1.39	1.23	0.91	1.27
9	Computers and Industrial Engineering	165	24	6.88	1.72	2.38	1.69	2.38
10	Manufacturing and Service Operations Management	106	14	7.57	2.64	1.58	1.45	2.69

In Table 18 shows the top twenty best authors for each database. The table includes the rank, author, the weighted metric NCIIW, the number of citations f and the number of publications n and relative frequency fr.

Table 18: Top twenty cited authors by NCIIW, stating number of citations f, number of publications n and relative frequency

Scopus Database						Web of Science Database					
Rank	Author	NCIIW	f	n	fr	Rank	Author	NCIIW	f	n	fr
1	Laporte Gilbert	428.08	253	17	14.88	1	Teresa Melo	376.00	275	2	137.50
2	Christiansen Marielle	331.27	199	12	16.58	2	Laporte Gilbert	366.00	214	16	13.38
3	Coelho Leandro C.	310.67	75	9	8.33	3	Nickel Stefan	329.00	275	2	137.50
4	Cordeau Jean-Francois	279.33	108	10	10.80	4	Saldanha-da-Gama F.	282.00	275	2	137.50
5	Ignaciuk Przemystaw	221.07	108	15	7.20	5	Coelho Leandro C.	276.00	69	9	7.67
6	Bartoszewicz Andrzej	193.43	108	16	6.75	6	Christiansen Marielle	266.53	161	10	16.10
7	Bertazzi Luca	189.34	126	9	14.00	7	Cordeau Jean-Francois	244.67	94	8	11.75
8	Andersson Henrik	175.33	109	5	21.80	8	Bertazzi Luca	174.12	132	9	14.67
9	Song Jin-Hwa	165.71	112	8	14.00	9	Song Jin-Hwa	153.46	100	8	12.50
10	Savelsbergh Martin	158.24	143	10	14.30	10	Archetti Claudia	150.00	88	6	14.67
11	Cannella Salvatore	155.68	87	7	12.43	11	Andersson Henrik	139.53	90	5	18.00
12	Ciancimino Elena	151.53	87	6	14.50	12	Li Xiuhui	131.00	131	1	131.00
13	Mete Huseyin Onur	145.60	91	1	91.00	13	Speranza Maria Grazia	125.85	127	8	15.88
14	Lokketangen Arne	135.60	142	5	28.40	14	Savelsbergh Martin	123.06	105	9	11.67
15	Archetti Claudia	131.00	86	5	17.20	15	Lokketangen Arne	120.93	124	6	20.67
16	Zhou Sean X.	130.47	67	10	6.70	16	Wang Qinan	118.63	132	2	66.00
17	Zabinsky Zelda B.	127.40	91	1	91.00	17	Zhou Sean X.	116.40	61	11	5.55
18	Moin Noor Hasnah	125.75	96	5	19.20	18	Chao Xiuli	103.02	61	11	5.55
19	Kjetil Fagerholt	125.50	61	6	10.17	19	Fagerholt Kjetil	101.75	49	6	8.17
20	Aghezzaf El-Houssaine	124.32	114	13	8.77	20	Hoff Arild	100.80	72	1	72.00

3.5 GROUPS OF RESEARCHERS

In this section we identified groups of the researchers that we considered important by their contributions. We assumed that the best groups also have the best researchers. For this reason the main factor in the selection of the groups are the best researchers as well as the connections that they have with other researchers and the number of the documents that they generated. The results obtained are showed in the summarized in the Figures 7 and 8.

Based on the research groups, the topics of interest of each group was identified and highlighted according with the publications of the most impact. Below, the in-

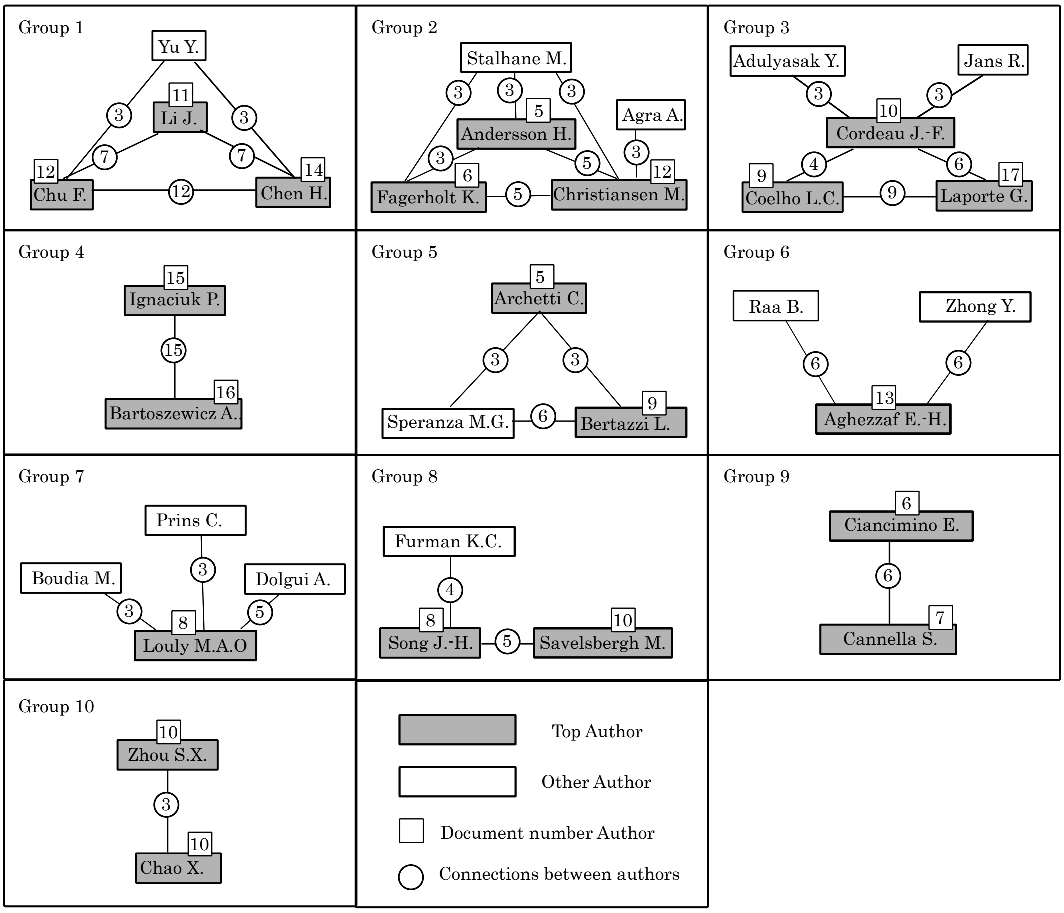


Figure 7: Groups of researchers in Scopus database. The groups 11, 12 and 13 were not highlighted

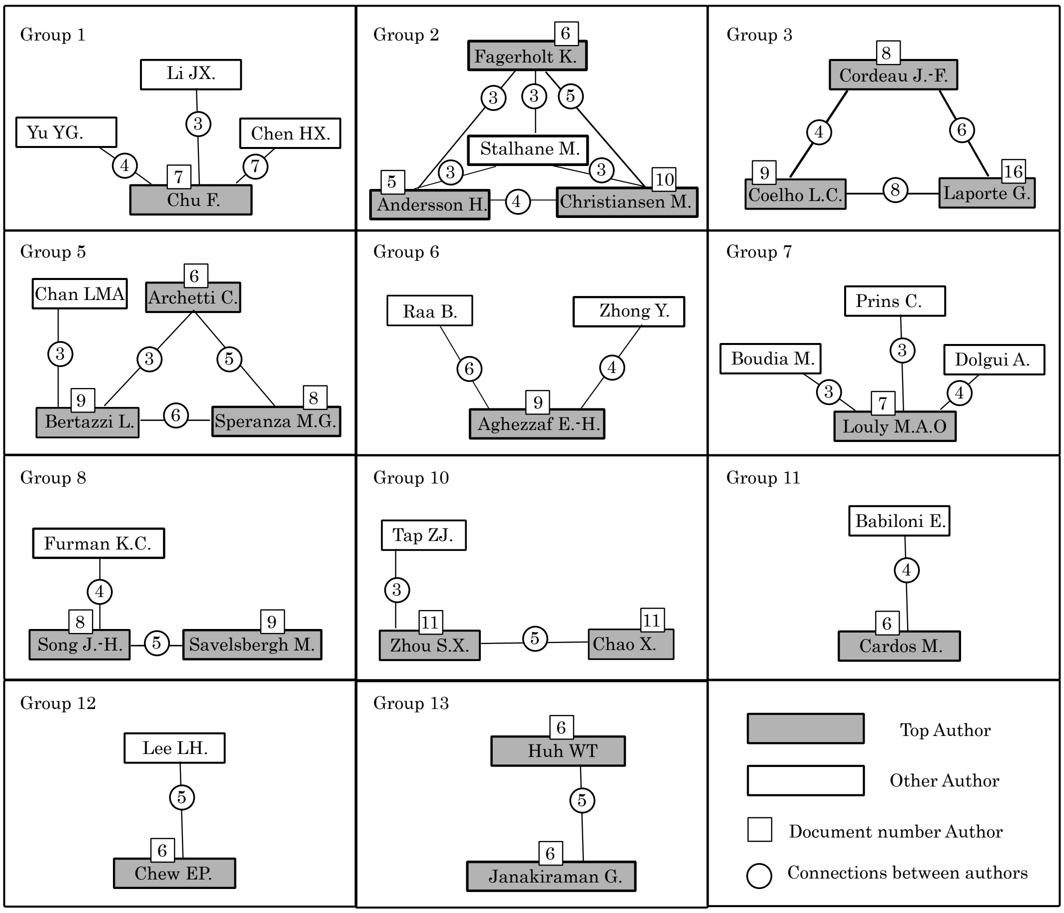


Figure 8: Groups of researchers in Web of Science database. The groups 4 and 9 were not highlighted

formation obtained is summarized. The groups of investigators identified in the two databases are:

Group 1: group with Chinese researches that involves severals institutions School of Management, University of Science and Technology of China, Department of Automatic Control, Beijing Institute of Technology, Rotterdam School of Management, Erasmus University Rotterdam, School of Management, Lanzhou University, Industrial Systems Optimization Laboratory, Charles Delaunay Institute of Technology of Troyes and Laboratoire IBISC, Université d'Evry. Their research interests include Performance evaluation of distribution strategies for the inventory routing problem [75], inventory routing problems with split delivery and stochastic demand study [129], also it is common that uses the Lagrangian relaxation method [128]. The most prominent researchers in this group are Feng Chu, Jianxiang Li, Haoxun Chen and YU Yugang.

Group 2: composed by Researchers Chisteansen, Andersson, Fagerholt and Lokketangen among others. The contributions of this group are generally affiliated to Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management: this group specializes in Maritime IRP problems or MIRP [4] and [31], also have worked Rolling horizon heuristics for solved problems of optimization [96].

Group 3: In this group, Coelho, Cordeau and Laporte were identified as the most outstanding researchers. This group of researchers based in Canada that involves the following institutions: HEC Montreal, Department of Logistics and Operations Management, CIRRELT and Laval University. We highlight of this group contributions on issues of heuristics for IRP in special Branch-and-cut [43], and [1], IRP with transshipments [41] and Stochastic and Dynamic IRP [42].

Group 4: Przemyslaw Ignaciuka and Andrzej Bartoszewiczb. They form a group of researchers based in Poland in the Institute of Technical and Institute of Automatic Control of the University of Lodz. The most notable contributions of this group are related to applications of sliding-mode control and discrete-time dynamical optimization in inventory ([60], [61], [62], [63], [64], [66] and [65]).

Group 5: In this group we found researchers of the Brescia University such as Bertazzi, Archetti and Speranza. This group specializes in foundations of IRP in special contributions in the formulation of problems [9], contributions in Stochastic IRP with stock-out [15] and the Branch-and-cut heuristic [8].

Group 6: El-Houssaine Aghezzaf, Birger Raa and Yiqing Zhong form a group of researchers based in Belgium in department of Industrial Management at Ghent University. The most notable contributions of this group are related to cyclical distribution plan and designing distribution patterns for long-term inventory routing [2] and [95].

Group 7: Alexandre Dolgui, Mohamed-Aly Louly and Christian Prins affiliated to University of Technology of Troyes in Charles Delaunay Institute. Their research interests include applications of combinatorial optimization to transportation and supply chain, production planning and stochastic models for inventory control [19] and [81].

Group 8: Savelsberg, Song Jin-Hwa, Doerner and Furman of an Interagency group that involves the institutions of University of Newcastle, School of Mathematical and Physical Sciences, Australia, Georgia Tech and ExxonMobil Research and Engineering Company. This group specializes in the maritime IRP problem or MIRP [114], stochastic inventory routing problem with direct deliveries [71], the heuristics branch-price-cut [55] and Variable neighborhood search [50].

Group 9: Salvatore Cannella, and Elena Ciancimino affiliated to faculty of engineering in the University of Palermo and Italian National Research Council (CNR). The most notable contributions of this group are related to increasing levels of shared information [20] and analysis of bullwhip effect in multi-echelon supply chain [33] and [34].

Group 10: Zhou, SX and Chao, XL conforms a group that involves severals institutions such as Department of Systems Engineering and Engineering Management The Chinese University of Hong Kong, Department of Industrial and Operations Engineering, University of Michigan. This group specializes in Stochastic inventory system [23], Finite ordering capacity [24], Optimal pricing decision and reverse logistic [132].

Group 11: A group of the Universidad Politécnica de Madrid constituted by Manuel Cardos and Eugenia Babiloni. The most important contributions of this group is to propose exact and approximate calculation of the cycle service level in periodic review inventory policies [21] and [12].

Group 12: E.P Chew and L.H. Lee are a group of researchers is the department of industrial and system engineering. National University of Singapore. They analyze of the impact of the random lead time in the supply chain [28] and also study the dynamic rationing problem for multiple demand classes with Poisson demands [58] and [27].

Group 13: Ganesh Janakiraman, Woonghee Tim Huh and Bijvank in Unites States group that include the institutions Department of Industrial Engineering and Operations Research, Columbia University, Stern School of Business, New York University. Their research interests focus on inventory theory where one of his interest topics is the (s, S) policy [56] and [57].

3.6 CONCLUSIONS

By detecting the lack of scientometrics studies in the subject of IRP, a study of this type is presented, which puts special emphasis on stochastic and dynamic version of the inventory routing problem with periodic revision of the inventory. For this propose, one search equation was used in which were chosen the keywords as well as subject areas the more interest for the IRP. The study collects 934 papers from Scopus and 720 from Web Science. The analysis focuses on number of publications, number of citations and detection of the groups of researchers. For the data processing, metrics to measure the productivity and quality were used such as Supply Chain Management (SCM), Author Position Method (APM) and Normalized Citation Impact Index (NCII). A new metric called Normalized Citation Impact Index weighted (NCIIW) that uses the criteria of the APM and NCII metric was used for the identification of the prominent authors. This metric allows relating the number of citations and the longevity of the publications to the author position. The most outstanding publications are identified, cited and referenced in this paper such as reviews, articles and conferences articles. Also, the journals and the conferences more used for the researcher to publish their results were highlighted. Based on the information of prominent authors, their connections with other authors and the number of their joint publications are used to identify the leading groups of rresearchers and their research and expertise. Finally, a journey through all tables in the paper allows us to know the state of the art of research of IRP, especially in stochastic, dynamic inventory routing problems with the periodic revision of the inventory as well as its trends and patterns.

Part II

METHODOLOGICAL ASPECTS

4 RESEARCH METHODOLOGY

The research methodology applies a positivist research paradigm in which, an hypothesis is tested by experiments. Thus, a conceptual design allows to define the hypothesis of the research from gaps in the literature and corresponding research questions. In order to relate the research methodology with the development methodology this chapter is divided in two parts.

In Section 4.1, the research methodology is explained by referring to seven aspects: i) the scope and significance of the problem, ii) the state of the science, iii) gaps in the literature, iv) variables under study, v) operationalization of variables, vi) instruments applied, data collection and vii) assumptions.

In Section 5 the development research methodology is explained by means of: i) preliminary procedure of development, ii) initial decomposition to the problem and iii) testing feasible solutions.

4.1 RESEARCH METHODOLOGY

4.1.1 *The scope and significance of the problem*

The coordination and the integration of the various components in the SC management have become critical in gaining competitive advantage. The price of the a product is the key for the competitiveness, but it is affected by the logistics cost which increase their cost. A greater proportion of the logistic costs correspond to the transportation and inventory processes. The inventory represents a proportion of net operating assets of approximately 37% in industry, 62% in distribution and 56% in retails.

In turn, the variability of the demand information affects the integration of the SC. The variability causes efficiency and efficacy losses influencing the decisions related to inventory control. In addition, it is important to note that the inventory control has to balance conflicting objectives due to two main reasons: i) economy of scale and purchasing batch size and ii) uncertainty in offer and demand with production and transportation lead time that inevitably creates the need for safety stock.

Besides that, the current models proposed in the literature are computationally efficient but have some difficulties at implementation time in the real world, due to the lack of flexibility incorporating additional constraints. In this context, it is mandatory to establish an optimal policy for the coordination of the flow of goods and services along the SC. This policy is applicable in the process of distribution and inventory in commercial relationships between companies. The overall costs are minimized as well as the uncertainty in the decisions. The decisions to be taken are related to how much to ship, when to ship and how to ship. These decision are taken in oder to decrease inventory and transportation cost, guaranteeing certain service level and adjusting to available resources.

Given the complexity of the IRP, studies are approached from instances of it, so models usually include only the variables of interest according to real-world problems. So, the basic problem to be considered have the following characteristics:

The SC is conform by one or many supplier(s) of product (It may be a factory or a provider) and several geographically dispersed retailers. The policy of inventory management called VMI is followed and only one actor is responsible of taking decisions. The study is focused in the analysis of inventory systems with stochastic demand and lead time. The product demand that is assumed unknown is from retailers to supplier, but its probability distribution is know and it is analyzed in discrete time. Each retailer has a capability of storage its own inventory with a maximum allowable limit. The vehicles used for transportation have limited capacity and the vehicle only does one travel by period.

The objective of analysis of the problem is to minimize the total cost of inventory and its distribution without causing stock-outs in the retailers, thus, it is necessary to determine the inventory level by period for each retailer and to determine strategies that minimize the associated costs for the distribution routes and inventory hosted.

4.1.2 *The state of the science*

According to the problem, the study objects that are Inventory Routing Problem and Inventory Control were identified. After conducting a review of the epistemological aspects of the objects of study, the following ontological elements that allow to conceptualize the problem were identified and delimited: information management, relationship of inventory policies with the demand information, demand and lead time modeling under uncertain demand, and optimization methods for single and multi suppliers.

With the aim of providing a global overview of the real situation of the problem related to the coordination of the inventory and its distribution, a state of the art is provided in [102] in which the objects of study were explained.

4.1.3 *Gaps in the literature*

The following describes the research purpose, research questions, research hypothesis and research contributions.

4.1.3.1 *Objectives*

The main objective and the specific objectives are defined considering the requirements that should fulfill a coordination policy in a SC.

Main Objective

The main objective is to coordinate inventory control and its distribution process in the SC, in order to reduce the costs and the uncertainty at the time of taking decisions based on demand. This coordination needs to fulfill the following characteristics:

- Reduce the time for the planning of a shipment.
- Increase the use of available resources.
- Select the retailers to be replenished.
- Determine when the chosen retailers are going to be replenished

- Determine how many units of product should be replenished to each retailer along the SC

The coordination will enable the measurement of the different factors influencing the process. Thus, the objective of the thesis concentrate in two points:

- Minimize the cost of inventory with low stock-out
- Minimize the transportation costs.

To achieve these main objectives, it is necessary to consider the following specific objectives:

Specific Objectives

Three specific objectives have been defined, the first is related to the modeling of the demand, the second is related to find areas for improvement and the last one about the test results.

- Relate the behavior of the SC with network flow optimization to implement solutions and methods of artificial intelligence.
- Establish specific points of improvement in studies cases that demonstrate the benefits of the coordination.
- Contrast the results in simulated contexts for different configurations to obtain conclusions with respect to the different techniques that can be employed in each of the cases.

4.1.3.2 *Research Questions*

The research questions identified are:

- What are the challenges in heuristic and metaheuristic methods to reduce computing time and find near-optimal solutions in inventory and distribution problems?
- How to dynamically configure inventory policies in the SC so that the effects of uncertain demand is reduced?
- How to reduce the effects of uncertain demand in SC so that the probability of stockouts decrease?
- How the SC can obtain a balance between different objectives that improve performance and allow comparable benefits for its different actors?

4.1.3.3 *Research Hypothesis According to the Impact Areas*

The set of hypotheses of the research work are grouped by areas of impact identified. Five hypothesis were defined and described next. The expected areas of impact to be analyzed with the proposed coordination are the following:

- To reduce the time for obtaining one near-optimal solution

- To join tactical and operational decisions
- To balance routes and cost
- To minimize transportation and inventory cost at the same time
- To enhance the service level

Reduce the time to obtain one near-optimal solution

One of the outcomes expected is to reduce the time to obtain a near-optimal solution. This is required due the dynamism of the system in real contexts where it is necessary to take decisions quickly. Therefore the following hypotheses have been made:

1. The IRP is a NP-Hard problem because it encompasses several classes of the well-known Multi-depot and Periodic Vehicle Routing Problem (VRP). Heuristics methods could be used to obtain a solution in a optimal time.
2. There is a near-optimal inventory level for each retailer that is better that predefined policies of replenishment which is possible to calculate dynamically. This is due to conventional forms to manage the inventory that presents problems related to the use of resources, load balancing of work and the priorities allocation which can be solved using VMI.

Join tactical operations with operational decisions

Another result is to achieve an integration of the type of decisions that can be taken in the SC, in this case, the tactical decisions are joined with operational solutions. This is due to the fact that when processes are integrated, more efficacy and efficiency is archived. These are the hypotheses that represent that idea:

3. An allocation scheme on two levels: the first one level performs the selection of retailers to server and the second one seeks the optimal route for distribution, this allows to join the tactical decision from inventory policies with the operational decisions of its transportations.

Balance the routes and cost

The balance in the SC is a characteristic that determines the maturity level, this balance can be observed in balanced routes and profits balanced for every actor. Two hypotheses are proposed:

6. Additional constraints in the SC will allow the searching of solutions with balanced routes, which are interesting for the industry in complex nets.
7. The correct integration between transportation cost and inventory cost in a optimized model will allow the balance in the profits for all the actors in the SC.

Reduce transportation and inventory cost at the same time

The expected outcome for the thesis is to reduce both transportation costs and inventory costs. These are analyzed from a joint perspective and taking into account the stochastic nature of the demand. One hypothesis have been considered:

8. The transportation cost and inventory cost can be reduced anticipating the demand, taking advantage of the stochastic demand presents in the SC and carefully selecting the solution in the Pareto Front.

Enhance the service level

The level of service is a parameter in this thesis. It is possible to reduce the probability of stockout in a retailer, although it can increase the total cost in the process under study. In this direction, the following hypothesis is defined:

9. The dynamic configuration of parameters for the inventory policy increases the probability to choose, for the retailers, replenishment with more risk of stock-out by each period.

4.1.3.4 *Contributions*

The contributions of the research are summarized in nine topics which are:

1. We review papers working with stochastic demand and stochastic lead times focusing on its stochastic and multi-depot aspects.

2. We identify critical factors for the performance of many logistics activities and industries.

3. We have show trends and patterns by means of tables in different topics of interest for the researches, which allows to know the state of the research on IRP field and specially in the topic of version stochastic, dynamic and the revision periodic of the inventory.

4. We have shown that studying the behavior of the demand and the lead time is essential in order to achieve a useful representation of the system to take proper decisions.

5. We propose new customer selection methods for a dynamic and stochastic inventory-routing problem

6. We perform a multi-criteria analysis of the solutions, comparing distribution versus inventory management

7. We perform a single criteria objective experiment on benchmark instances from the literature for single and multi suppliers.

8. We develop the Inventory Replenishment and Customer Selection Policies Algorithm (IRCSPA) algorithm for solve the DSIRP with single supplier and Hybrid Genetic Algorithm with Network Flow Fitness (HGANFF) for solver the Multi Depot Dynamic Stochastic Inventory Routing Problem (MDDSIRP).

9. Our methods yield improvements over a competing algorithm on average.

4.1.4 *Variables under study*

The independent variables, the dependent variables and the intervening variables are identified and are defined next.

4.1.4.1 *Independents Variables*

Three variables have been identified. These are inventory policies, inventory constraints and transportation constraints. A description of each one is shown next:.

Inventory policies

An inventory policy is a standard set of rules, boundaries and guidelines that provide the framework for an organization to make better informed and timely decisions on which stock to purchase or manufacture, how much stock to purchase or manufacture and where to store and distribute to customers.

Inventory constraints

Inventory constraints refer to so-called cumulative resources, which can store a single or several different products and have a prescribed minimum and maximum inventory, where the inventory is depleted and replenished over time.

Transportation constraints

A set of the transportation constraints can be defined to formulate the problem, this depends on the instance to be analyzed. The most common constraints are related to the number of the vehicles, capacity and time windows.

4.1.4.2 *Dependents Variables*

Four variables are identified and described next.

Inventory levels

A inventory level is defined as the current amount of a product that a business has in stock. The inventory level and sales rate of a product will be used by a typical inventory manager to determine the optimal time for either producing more, if they are managing a manufacturer's warehouse, or to order more if the product is being stored as stock at a retail store.

The inventory levels depends on the inventory level at previous period I_i^{t-1} the quantity of inventory that arrives in the current period q_i^t, the current demand d_i^t and the lost current demand l_i^t. The Equation 1 shows this relation between the variables.

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t + l_i^t \tag{1}$$

Inventory Cost

Inventory costs are the costs related to storing and maintaining its inventory over a certain period of time

Transportation Cost

The expenses in which a company incurs when it transfers its inventory or other assets to another location.

Computation time

Calculation time naturally depends on the used computer. Calculation time is most of time only relevant for large data sets and most relevant is the speed of growth of calculation time dependent on the size of the data set [123]. Our algorithms are executed in typical working stations available in any office with normal conditions. The is calculate as the time that take the algorithm to give an acceptable solution. Through of experiments in instances of different sizes is possible to observe the growth of calculation time.

4.1.4.3 *Intervening variables*

An intervening variable facilitates a better understanding of the relationship between the independent and dependent variables when the variables appear not to have a definite connection. In the inventory and its distribution, VMI policy can intervened between the independent and dependent variables and reduce costs.

Vendor Managed Inventory

VMI is a family of business models in which the buyer of a product (retailer) provides certain information about inventory to a supplier and it takes full responsibility for maintaining an agreed inventory of the material, usually at the buyer's consumption location (usually a store). A third-party logistics provider can also be involved to make sure that the buyer has the required level of inventory by adjusting the demand and supply gaps.

4.1.5 *Operationalization of variables*

The models and methods used in this work are based in a structure of costs, which is the most popular in industrial applications. The costs considered are basically two: host inventory cost and lost sales for stock-outs, in this thesis called lost demand. Eventually problems of production planning are studied added to IRP called. Also, in the models considered it is assumed that basic conditions for the inventory are given such as: probability distributions are know, lead times, service requirements, inventory cost and for stock-outs along others. The same for transportation.

The metrics used in this work are taken of study [46] that compiles and synthesizes important elements related to SC performance measurement. The performance measures are grouped by costs, flexibility and dynamism measures. Next, these metrics are described:

Cost measures

The decision maker must evaluate all aspects that are incorporated in each link of the chain. One effective measure of the performance of the SC is the cost. The costs described below were those considered in this thesis

- Distribution Costs: cost or expense incurred in moving goods from the supplier to retailers. Also called distribution expenses.
- Inventory Costs: the costs related to storing and maintaining its inventory over a certain period of time.
- Total Costs: Total cost refers to all the costs incurred to manufacture a product. This includes costs of raw materials, labor, transportation, distribution, marketing, administrative, overhead, as well as fixed costs such as acquisition of property, machinery, and other longterm investments. however in this work only inventory (with lost demand) and transportation cost are consider.

Flexibility measures

- Capacity Utilization: every company has capacity limitations on equipment; the extent to which the company utilizes their equipment is capacity utilization.
- Delivery Flexibility: This measure assesses the company ability to schedule delivery dates. If the company delivers the goods or products prior to an anticipated date, the customer will likely be satisfied.

Measuring dynamism

According to [92] citing [59] and [82] different problems (or instances of a same problem) can have different levels of dynamism, which can be characterized by two dimensions: the frequency of changes and the urgency of requests. The former is the rate at which new information becomes available, while the latter is the time gap between the disclosure of a new request and its expected service time.

4.1.6 *Instruments applied and data Collection*

According to [102], dealing with complex problems, such as the IRP, it is common to place a set of instances or testing problems available for others. In the specific case of routing vehicles, it is also possible to find instance sets proposed by researchers belonging to CIRRELT, SCL, OR@Brescia and Logistics Management Department of Helmut-Schmidt-Universitat, online [1] [2] [3] [4].

This instances are called Benchmark Instances. In this research the Coelho instances are used.

The results obtained in this research are tested with the best-solutions of the benchmarking. As the results are obtain with metaheuristics methods and hybrid algorithm.

4.1.7 *Assumptions*

- There is a centralized decision model that allows to only one "decision maker" to realize optimization actions in SC.

1 http://www.leandro-coelho.com/instances/
2 http://www.tli.gatech.edu/research/casestudies/irp2/
3 https://sites.google.com/site/orbrescia/home
4 http://www.hsu-hh.de/logistik/

- The decision maker has access to historic demand information of each retailer. This information relates specifically to the mean and variance by time unit. Also, the decision maker knows the probability distributions of the orders of retailers.
- The demand data are analyzed in time discrete intervals, generally in days.
- The probability of distribution of the leadtime is known.

5 DEVELOPMENT METHODOLOGY

A version of this work was presented and published in proceeding abstracts of the International Conference on Operational Research KOI 2014 by Raúl Roldán, Rosa Basagoiti and Enrique Onieva which has the title A Framework in the Formulation and Solution of Inventory Routing Problems ([100]). The abstract is presented below.

Researchers who investigate in fields relate with optimization problems in Supply Chain (SC), in special those that involve the process of inventory and its distribution, find difficulties to relate the knowledge areas such as operation research and computer science, organizing the procedure and evaluating the solutions obtained. After analyzing this problem, a simple framework has been developed to use in the searching of near-optimal solutions in the field of Inventory Routing Problems (IRP). In this paper this framework is described in detail, and all the phases to follow are introduced step by step. Although some of these phases can be extended for other type of optimization problem in the SC, the literature of this study is focused in IRP. This field has been chosen due to its importance in the real world, and its great relevance in the literature. The use of benchmark instances for evaluating results is highlighted and these instances are organized according the concrete problem for which are used. Also, some key elements to face the problem are presented such as the information management, relationship of inventory policies with the demand information, demand and lead time modeling and optimization methods. These elements are organized and classified for use case.

The methodology followed in this work is based on an incremental and iterative development. In [35] the terms incremental and interactive are defined. In an iterative development rework (to make continuous changes in order to improve) scheduling strategy is used, where time is set aside to revise and improve parts of the system (see Figure 9). In an incremental development, staging and scheduling strategy is used, in which various parts of the system are developed at different times or rates and integrated as they are completed (see Figure 10). The idea behind these methods is to develop a system through repeated cycles (iterative) and in smaller portions at a time (incremental). It is important to notice that neither strategy presupposes, requires, or implies the other. It is possible to use both of them or alone.

The unified process, iterative and incremental, can be divide into four phases:

Figure 9: An iterative model to develop systems

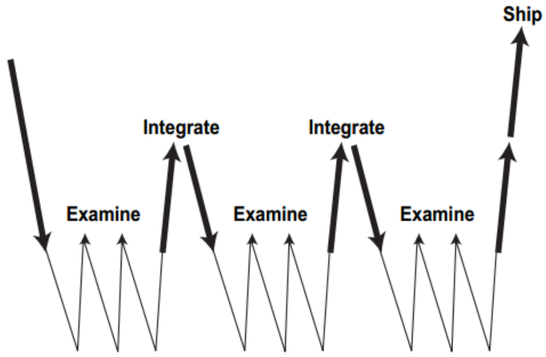


Figure 10: Iterative and incremental development methods put together

1. To define the problem: for this phase, the requirements of the system, such as, the transportation net, routes, transportation and inventory constraints are needed. The knowledge of the experts and personal experiences are important to achieve a adequate configuration of the problem. All the information about the problem allows to build an instance. An instance is a set of characteristics of the problem that will be analyzed, which are determined by the definition and constraints of the system. New characteristics are added to this instance, as previous stages are overcomed, and a new instance of the problem is formulated.The definition of the problem needs the identification of the variables of the system and this is a complete task to be integrated in the development.

2. To use instance: as the system requirements are captured, it is necessary to choose some benchmark instances. The benchmark instances to select depends on the instances of problem analyzed, then these need to be changed according to the definition phase. Sometimes, instances that have been used by others researchers can be used. For another situations it is necessary to create them. In this work, a mathematical model is defined to test the selected benchmark instances and the consistency of the identified variables.

3. To design an algorithm: in order to obtain a near-optimal solution for the instances of the problem, an algorithm is designed and parametrized. The algorithm is tested and its parameters modified according to the solutions required. Generally, the problem is decomposed in parts according the state of the art, technological background and the technique to be used. General parts identified so far are: a)selecting or clustering the retailers for one or many suppliers, b)loading the vehicle or fleet of vehicles available, c)calculating the near-optimal solution and d)calculate the inventory, transportation and lost demand costs. Many versions of the algorithm are implemented and each version is an incremental development that can be integrated in the system.

4. To evaluate the solution: the algorithm is tested with the benchmark instances and previous research analysis results. Exact models are developed for better understanding the quality of the results. If instances have already been tested by other researchers the results obtained will be compared to their results.

These phases are repeated according to what is required in the development.The integration of all these parts with the incremental and iterative development, allows to propose a model for coordination of the IRP.

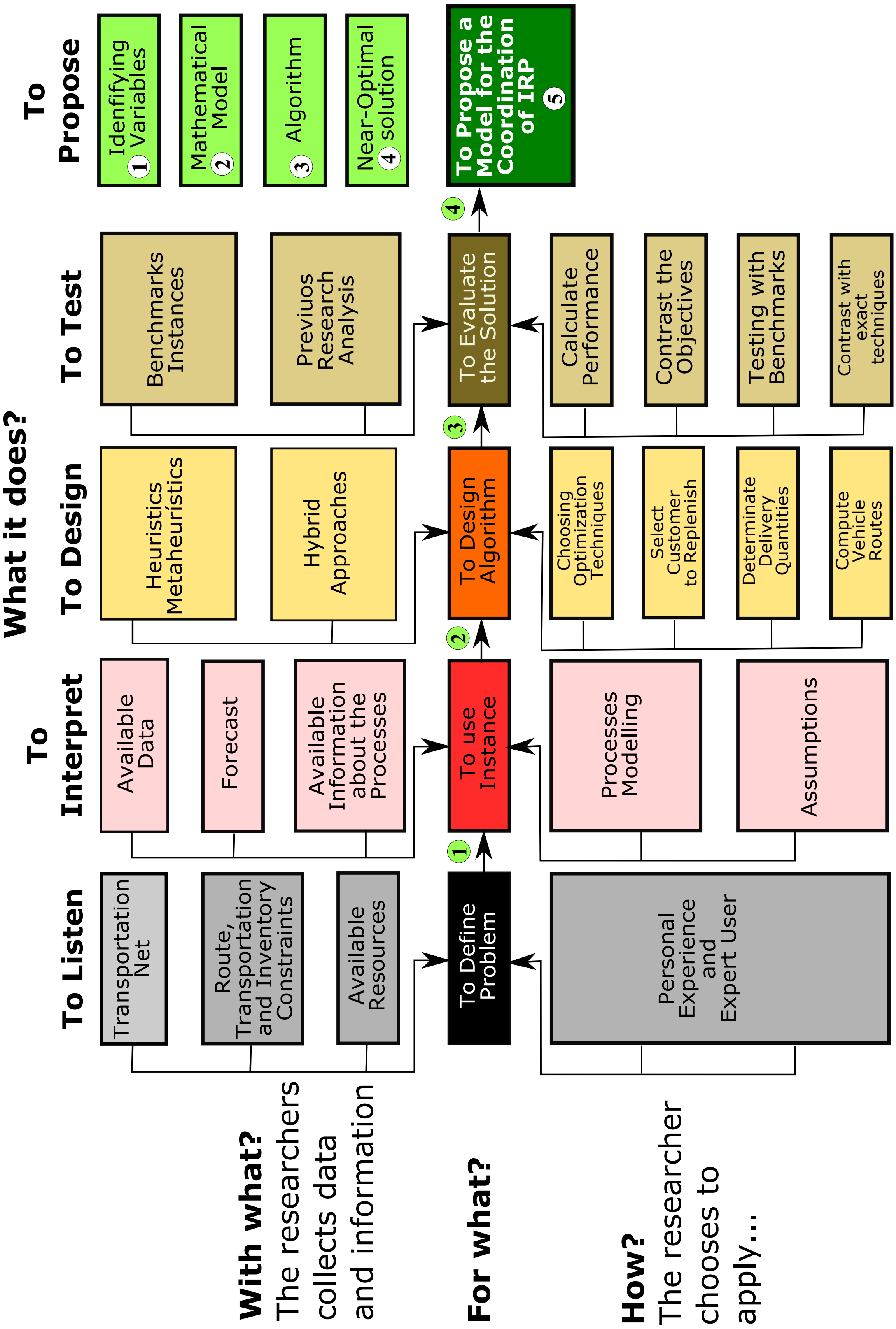


Figure 11: Development methodology

Figure 11 depicts the general procedure we have followed in the development of this work. This is explained from the point of view of the actions that should follow a researcher interested in addressing the field of the IRP problems.

Next, a initial decomposition of the problem is provided in order to create parts that can be solved independently and then integrated together. The procedure described below was used in Chapter 7. Also, a preliminary procedure for the development was presented in the Chapter 8.

5.1 INITIAL DECOMPOSITION OF THE PROBLEM

Integrating the inventory and its distribution, the Inventory Routing Problem arises, which is an NP-hard problem. This research is focused on obtaining the near-optimal solutions to the problem instead of obtaining its optimal solution. Therefore, heuristics and metaheuristics in conjunction with intelligence artificial methods are applied.

The procedure of solution is composed of 4 phases:

- Selecting and clustering retailers: the retailers are selected by means of inventory policies, this guides the optimization process
- To Load the Vehicles: in order to load the vehicle it is necessary to take account of the capacity constraints. In case that some retailers cannot be served, these should be prioritized for the next time period.
- To Calculate Routes: A traveling salesmen problem is used to determine the near-optimal route that allows to minimize transportation costs.
- To Replenish Inventory levels: The inventories levels of the clients are replenished and the total costs calculated.

5.2 PRELIMINARY PROCEDURE TO THE DEVELOPMENT

A initial procedure to analyze the SC have been considered. This procedure is composed of the following stages:

- Optimize statistic instances with the following characteristics: one product, one or many supplier(s), one or fleet of vehicle(s) with finite capacity and infinity capacity in the supplier. Results can be observed in the outcomes variables such as: inventory cost, transportation cost and lost demand cost. The level of service is also considered.
- Incorporate the historic data to reduce the bull-wild effect and compare the improvement. Measure the quality of the historic data by means of relative error and absolute error.
- Calculate the optimal inventory level for each retailer and measure the impact on other indicators.
- Incorporate more vehicles controlling their capacity and compare the results
- Incorporate more suppliers and compare the results
- Compare the results with the results provided by exact methods.

5.3 TESTING FEASIBLE SOLUTIONS

Feasible solutions are tested by means of the comparison between the near optimal solution benchmark instances with near-optimal solution obtained with the proposed optimization algorithm.

Part III

BACKGROUND FORMULATIONS

6 BACKGROUND FORMULATIONS FOR A DSIRP

6.1 INTRODUCTION

In this chapter we perform a review of the most important mathematical models that are foundation to face the IRP. In order of complexity, it is possible to decompose the IRP in different sub-problems. One of them is motivated by the need of selecting the best route to go from one deposit to a set of clients. This problem is known as the Travelling Salesman Problem (TSP), a classical combinatorial optimization problem. TSP details may be consulted in Matai, Mittal, and Singh [85]. Since it is also important to transport products, it is necessary to add restrictions to routes to be fulfill by vehicles, then this problem becomes a VRP. The VRP, variants and features can be consulted in Toth and Vigo [119]. Additionally, when the levels of consumption of clients and suppliers in order to maintain a continuous replacement are considered, an IRP is generated. An IRP fulfills three aims: i) establish the optimal inventory levels, ii) plan the volume and number of shipments and iii) ensure that deliveries suit the requirements of each product.

In Section 6.2 we present four TSP mathematical formulations. The versions, basic TSP, mTSP, Multi-Supplier TSP and Multi-supplier mTSP are presented. In Appendix A the dataset used are described. The following problem, the VRP, in Section 6.3 is presented. For the VRP problem 5, the variants, are chosen, CVRP (with homogeneous and heterogeneous fleet), VRPTW, and Multi-supplier and Periodic VRP. The dataset used can be found in Appendix B. For last, the mathematical model for IRP problems are presented in Section 6.4. In this section, the versions basic IRP, Multi-period and Multi-supplier and Multi-period IRP are formulated. The dataset for IRP, in Appendix C are described. For all Models are tested in CPLEX.

6.2 TRAVELLING SALESMAN PROBLEM

The TSP is the first problem which reference should be made to address the optimization of logistic processes of inventory and distribution. In a SC), the TSP can be used to find the optimal route to visit a set of geographically distributed retailers starting from and returning to the depot. The objective is to minimize the travel time or the total travel costs. Four of the main mathematical models of linear programming to this problem are reviewed from Section 6.2.1 to Section 6.2.4. The different types of TSP are showed in the Figure 12 by means of a example with 10 retailers.

6.2.1 *Basic TSP*

Given a set of retailers n, and the cost of travel c (or distance) between each possible pairs ij, the TSP, is to find the best possible way of visiting all the retailers and returning to the starting supplier with one only vehicle, that minimizes the travel cost (or travel distance) z.

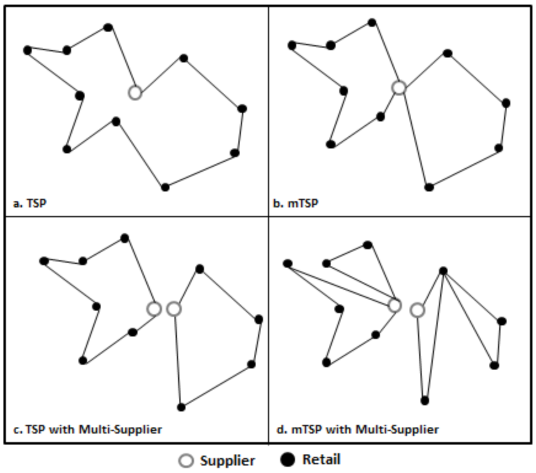


Figure 12: Feasible solutions for different types of TSP

The problem is defined on a graph $G = (V; A)$ where $V = 1, ..., m + n$ are all elements of the vertex set, with m and n as the number of suppliers and retailers respectively and A is the arc set, for all pair $(i, j) \in V$ where $i \neq j$. The vertex contains the supplier set D, where $D = 1, ..., m$ and the retailers set V' where $V' = m + 1, .., m + n$.

In the first model we consider one supplier ($m = 1$) and several retailers. For all node pairs (i, j), let $x_{i,j}$ be a binary variable, taking the value 1 if and only if the vehicle from node i to node j. Also, for $i \in V'$ let u_i be a continuous variable representing the position of node i in the tour. Thus the problem is to find variables x_{ij} and u_i where $i, j \in V$ and $m + n$ are all the nodes that should be traveling that minimize of the transportation cost c_{ij} given by the following objective function:

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{2}$$

subject to

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \tag{3}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V \tag{4}$$

$$u_i - u_j + |V| x_{ij} \leqslant |V| - 1 \quad \forall i, j \in V' \tag{5}$$

$$1 \leqslant u_i \leqslant |V| \qquad \forall i \in V' \tag{6}$$

$$x_{ij} = 0, 1 \quad \forall i, j \in V \tag{7}$$

The Constraint 3 guarantees that only one arc starts on the node i. The Constraint 4 guarantees that only one arc finishes on the node j. Together the Constraints 3 and 4 ensure that the vehicle arrives at and departs from each node exactly once. The Constraint 5 avoids sub-routes and together with the bounds of Constraint 6 ensure that each retail is in a unique position. Finally, Constraint 7 is the bounds definition of the binary variable x ensures that the only values are 0 and 1.

6.2.2 *Multi-Travelling TSP*

For this problem, several routes are found, one for each tour T such that each tour originates and ends at the supplier where it began (see Figure 12 part b). Similar to the first model we consider one supplier ($m = 1$) and several retailers where each retailers i, $i \in V'$ is visited exactly once, and total cost are minimized.

For all node pairs (i, j), let $x_{i,j}$ be a binary binary, taking the value 1 if and only if the vehicle from node i to node j. Also, for $i, j \in V'$ let y_{ij} be a continuous variable representing the position of node i in the tour. Thus, the problem consists in finding variables x_{ij} and y_{ij} where $i, j \in V$ and i, j is the number of the nodes that should be traveling. In this model, T is a parameter that should be defined beforehand. The aim is to minimize the transportation cost c_{ij} for all routes, according to the number of tours defined, given by the following objective function:

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{8}$$

subject to

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \tag{9}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \tag{10}$$

$$\sum_{i \in V} x_{i1} = T \tag{11}$$

$$\sum_{j \in V} x_{1j} = T \tag{12}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V} y_{ji} = 1 \quad i \in V' \tag{13}$$

$$y_{ij} \leqslant (|V| + 1 - T) x_{ij} \quad \forall i, j \in V \tag{14}$$

$$x_{ij} = 0, 1 \quad \forall i, j \tag{15}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \tag{16}$$

The Contraint 9 and 10 are equal to the ones in the previous model. The Constraints 11 and 12 assign to the supplier the number of travels that should perform according to parameter T given beforehand. From 13 it allows that at least one arc leads out from any retailer. It is important to note that all tours include to the supplier.

For route balancing, it is possible to add the Equation 17, where L and U are the lower and upper bounds on the number of retailers traveled.

$$x_{i1} L \leqslant y_{i1} \leqslant x_{i1} U \quad \forall i \in V' \tag{17}$$

6.2.3 *Multi-Supplier TSP*

In Multi-supplier problems, we have m suppliers which should be used as starting points to travel a set of retailers and each retail should be visiting only supplier. Also, the route starts and ends in the same supplier. Thus, the problem consists in finding variables x_{ij} and y_{ij} where $i, j \in V$ and i, j is the number of the nodes that should be traveling. The aim is to minimize the transportation cost c_{ij} for all routes using the following objective function

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{18}$$

subject to

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \tag{19}$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \tag{20}$$

$$\sum_{j \in V} y_{ij} = 0 \quad \forall i \in W \tag{21}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V} y_{ji} = 1 \quad i \in V' \tag{22}$$

$$y_{ij} \leqslant (|V| - 1) x_{ij} \quad \forall i, j \in V \tag{23}$$

$$x_{ij} = 0, 1 \quad \forall i, j \tag{24}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \tag{25}$$

The Constraint 19 and 20 are equal than the previous models. The Constraint 21 and 22 assigned to supplier the number of travels that should perform according to parameter T given beforehand. From 23 and on, it allows that, at least, one arc leads out from each one of the retailers. It is important to note that all tours include the supplier.

As the previous model, the equation 26 may be used if it is required to balance the routes that correspond to each supplier, where L and U are the lower and upper bounds on the number of traveled retailers. This equation is applied to each supplier.

$$x_{i1}L \leqslant y_{i1} \leqslant x_{i1}U \quad \forall i \in V', j \in W \tag{26}$$

6.2.4 *Multi-Supplier and Multi-Travelling TSP*

In Multi-supplier problem we have m suppliers which may be used as starting point to create the routes. For each supplier, multi-tours are allowed and these tours should start and ends in the i-th supplier; c_{ij} will be the traveling cost from node i to node j .

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{27}$$

subject to

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \tag{28}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \tag{29}$$

$$\sum_{j \in V} x_{ij} = T \quad \forall i \in D \tag{30}$$

$$\sum_{j \in V} x_{ji} = T \quad \forall i \in D \tag{31}$$

$$L \leqslant \sum_{i \in V} y_{ij} \leqslant U \quad \forall j \in D \tag{32}$$

$$x_{ij}LT \leqslant y_{ij} \leqslant x_{ij}UT \quad \forall i \in V, \forall j \in D \tag{33}$$

$$\sum_{i \in V} y_{ij} == |V| \quad \forall j \in D \tag{34}$$

$$\sum_{j \in V} y_{ij} = 0 \quad \forall i \in D \tag{35}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V} y_{ji} = 1 \quad i \in V' \tag{36}$$

$$y_{ij} \leqslant (|V| - 1)x_{ij} \quad \forall i, j \in V \tag{37}$$

$$x_{ij} = 0, 1 \quad \forall i, j \tag{38}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \tag{39}$$

The Constraint 28 and 29 are equal than the previous models. The Constraint 30 and 31 assigned to supplier the number of travels that should perform according to parameter T given beforehand. From 32 and on, it allows that, at least, one arc leads out from each one of the retailers. It is important to note that all tours include the supplier.

For route balancing, it is possible to add the Equation 36, where L and U are the lower and upper bounds on the number of traveled retailers.

As the previous model, the equation 26 may be used if it is required to balance the routes that correspond to each supplier, where L and U are the lower and upper bounds on the number of traveled retailers. This equation is applied to each supplier.

6.3 VEHICLE ROUTING PROBLEM

The solution of a VRP calls for the determination of a set of routes, each performed by a single or set of vehicles that start and end at its own supplier, such that all the requirements of the retailers are fulfilled, all the operational constraints are satisfied, and the global transportation cost is minimized. In this section, we describe the typical characteristics of the routing and scheduling problems by considering their main components (road network, retailers, suppliers and vehicles). We take into account some different operational constraints that can be imposed on the construction of the routes, and the possible objectives to be achieved in the optimization process.

We first describe the Capacitated Vehicle Routing Problem (CVRP), which is the simplest and most studied member of the family, then we introduce the Vehicle Routing Problem with Time Windows (VRPTW), Multi-Vehicle VRP with homogeneous fleet , Multi-Vehicle VRP with heterogeneous fleet , Multi-Vehicle and Multi-Supplier VRP with homogeneous fleet and Multi-Vehicle and Multi-Supplier VRP with heterogeneous fleet.

6.3.1 *Homogeneous fleet CVRP*

In this model, a non-negative load d_i associated with each node i, where $i \in V'$, is given to be delivered by K vehicles in an equal number of routes. It is necessary to find T routes of minimal total cost that leave a supplier $m = 1$, visit each node only once, and return to the supplier. The vehicles start to travel with the total load of the retail's route, and in each stop j, each vehicle is unloaded by the extra load d_j. There is a limit Q on vehicle capacity (homogeneous fleet) and the amount to be delivered in each tour cannot exceed this limit.

The objective is to minimize the total transportation cost for the routes. Also, to minimum the number of routes for the set of identical vehicles K. It is noted that the accumulated service up to any node does not exceed a positive number Q (vehicle capacity).

For the one supplier $(m = 1)$ specific case, the formulation can be reduced to the following model. The problem will be to find variables x_{ij}, y_{ij} and K, where $i, j \in V$ and i, j is the number of the nodes that vehicles should be traveling that minimize of the transportation cost c_{ij} given by the following objective function:

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{40}$$

subject to

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \tag{41}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \tag{42}$$

$$\sum_{j \in V'} x_{1j} = K \tag{43}$$

$$\sum_{j \in V'} x_{j1} = K \tag{44}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V} y_{ji} = d_i \quad \forall i \in V' \tag{45}$$

$$d_j x_{ij} \leqslant y_{ij} \quad \forall i \neq j \in V \tag{46}$$

$$y_{ij} <= (Q - d_i) x_{ij} \quad \forall i \neq j \in V \tag{47}$$

$$x_{ij} = 0, 1 \quad \forall i, j \in V \tag{48}$$

It is important to note that it is possible to fix the number of vehicles beforehand changing the variable K by a constant.

Another extension of this delivery problem is the case in which the number of vehicles K is not given beforehand, and there is an extra fixed cost ck associated with each additional vehicle used for delivery. Thus, the objective function referred to Equation 40 should be changed by Equation 49 and the constraint referred to Equation 43 and 44 should be replaced by 50 .

$$\text{minimize} \sum_{i \in V'} \sum_{j \in V} c_{ij} x_{ij} + \sum_{j \in V} (c_{1j} + ck) x_{1j} \tag{49}$$

$$\sum_{j \in V} x_{1j} = \sum_{j \in V} x_{j1} \tag{50}$$

6.3.2 *Heterogeneous fleet CVRP*

In this version of the VRP, a fleet a vehicles with different characteristics is used. Vehicle type h has a capacity Q_h, and a fixed cost P_{mh}, for using vehicle type h from the supplier $m = 1$. There is a limit T_{ih} on the number of vehicle type h which may originate from the supplier; c_{ijh} is the traveling cost from node i to node j using vehicle type h. We assume that a node is serviced by just one vehicle.

$$\text{minimize} \sum_{i \in V'} \sum_{j \in V} \sum_{h \in H} c_{ijh} x_{ijh} + \sum_{j \in V'} \sum_{h \in H} (c_{1jh} + P_{ih}) x_{1jh} \tag{51}$$

subject to

$$\sum_{i \in V} \sum_{h \in H} x_{ijh} = 1 \quad \forall j \in V' \tag{52}$$

$$\sum_{j \in V} x_{ijh} - \sum_{j \in V} x_{jih} = 0 \quad \forall h \in H, \forall i \in V \tag{53}$$

$$\sum_{j \in V'} x_{ijh} \leqslant K_{ih} \quad \forall h \in H, \forall i \in D \tag{54}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V'} y_{ji} = d_i \quad \forall i \in V' \tag{55}$$

$$y_{ij} \leqslant \sum_{h \in H} Q_h x_{ijh} \quad \forall i \in V', \forall j \in V, \forall h \in H \tag{56}$$

$$x_{ijh} = 0, 1 \quad \forall i, j \in V, \forall h \in H \tag{57}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \in V \tag{58}$$

6.3.3 *Multi-Supplier with Homogeneous fleet CVRP*

Several depots m, are available to service retailers in the Multi-depot VRP (MDVRP), k representing the number of vehicles avaibles at each depot. In this cases, the vertex contains the supplier set D, where $D = 1, ..., m$ and the retailers set V' where $V' = m+1, .., m+n$.

$$\text{minimize} \sum_{i \in V'} \sum_{j \in V} c_{ij} x_{ij} + \sum_{i \in D} \sum_{j \in V'} (c_{ij} + P_i) x_{ijh} \tag{59}$$

subject to

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V' \tag{60}$$

$$\sum_{j \in V} x_{ij} - \sum_{j \in V} x_{ji} = 0 \quad \forall i \in V \tag{61}$$

$$\sum_{j \in V'} x_{ij} \leqslant K_i \quad \forall i \in D \tag{62}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V'} y_{ji} = d_i \quad \forall i \in V' \tag{63}$$

$$y_{ij} \leqslant \sum_{h \in H} Q_h x_{ijh} \quad \forall i \in V', \forall j \in V, \forall h \in H \tag{64}$$

$$x_{ijh} = 0, 1 \quad \forall i, j \in V, \forall h \in H \tag{65}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \in V \tag{66}$$

6.3.4 *Multi-Supplier with heterogeneous fleet CVRP*

In Multi-supplier delivery problems, we have m suppliers which may be used as starting points for routes. One extra restriction exists: a tour will always return to the same supplier from which is started. Different types of vehicles may be used for performing the deliveries. Truck type h has a capacity Q_h, and a fixed cost P_{mh} for using vehicle type h from the $m-th$ supplier; there is a limit T_{ih} on the number of vehicle type h which may originate from the i-th supplier; c_{ijh} is the traveling cost from node i to node j using vehicle type h. We assume that a node is serviced by just one vehicle.

$$\text{minimize} \sum_{i \in V'} \sum_{j \in V} \sum_{h \in H} c_{ijh} x_{ijh} + \sum_{i \in D} \sum_{j \in V'} \sum_{h \in H} (c_{ijh} + P_{ih}) x_{ijh} \tag{67}$$

subject to

$$\sum_{i \in V} \sum_{h \in H} x_{ijh} = 1 \quad \forall j \in V' \tag{68}$$

$$\sum_{j \in V} x_{ijh} - \sum_{j \in V} x_{jih} = 0 \quad \forall h \in H, \forall i \in V \tag{69}$$

$$\sum_{j \in V'} x_{ijh} \leqslant K_{ih} \quad \forall h \in H, \forall i \in D \tag{70}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V'} y_{ji} = d_i \quad \forall i \in V' \tag{71}$$

$$y_{ij} \leqslant \sum_{h \in H} Q_h x_{ijh} \quad \forall i \in V', \forall j \in V, \forall h \in H \tag{72}$$

$$x_{ijh} = 0, 1 \quad \forall i, j \in V, \forall h \in H \tag{73}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \in V \tag{74}$$

6.3.5 *Heterogeneous fleet VRPTW*

According with Toth and Vigo [119], the VRPTW is an important problem occurring in many distribution systems. The VRPTW can be defined as follows. Each retail can be serviced only within a specified time interval or time window and a set A of arcs with nonnegative weights, d_{ij}, and with associated travel times, t_{ij}. The travel time, t_{ij}, includes a service time s_i at node i, and a vehicle is permitted to arrive before the opening of the time window, and wait at no cost until service becomes possible, but it is not permitted to arrive after the latest time window. The objective is to minimize the total transportations cost of the routes. Also, the minimum number of routes, for a set of identical vehicles K, is found, in this manner each node is reached within its time window and the accumulated service up to any node does not exceed a positive number Q (vehicle capacity). The formulation can be reduced as following:

$$\text{minimize} \sum_{h \in H} \sum_{i \in V} \sum_{j \in V} c_{ijh} x_{ijh} \tag{75}$$

subject to

$$\sum_{h \in H} \sum_{j \in V'} x_{jih} = 1 \quad \forall i \in V' \tag{76}$$

$$\sum_{j\in V'} x_{1jh} = 1 \quad \forall k \in K \tag{77}$$

$$\sum_{j\in V'} x_{ijh} - \sum_{j\in V'} x_{jih} = 0 \quad \forall h \in H, \forall i \in V' \tag{78}$$

$$\sum_{i\in V'} \sum_{j\in V'} d_i x_{ijh} \quad \forall h \in H \tag{79}$$

$$y_j h - y_i h \geqslant s_i + tij - M(1 - xijh) \quad \forall i,j \in V', h \in K \tag{80}$$

$$E_i \leqslant y_i h \leqslant L_i \quad \forall h \in H, \forall i \in V' \tag{81}$$

$$x_{ij} = 0, 1 \quad \forall i,j \in V, \forall h \in H \tag{82}$$

$$y_{ik} \geqslant 0 \quad \forall h \in H, \forall i \in V' \tag{83}$$

6.3.6 *Multi-Supplier and periodic CVRP*

A time dimension is introduced in the Periodic VRP (PVRP) as route planning is to be performed over a horizon of t periods. Multi Depot and Periodic Vehicle Routing Problem (MDPVRP) extends the two previous problem setting, asking for selection of a depot for each retail, with services in different periods to the same retail being required to originate at the same depot.

$$\text{minimize} \sum_{t\in P'} \sum_{i\in V'} \sum_{j\in V} \sum_{h\in H} c_{ijh} x_{ijht} + \sum_{t\in P'} \sum_{i\in D} \sum_{j\in V'} \sum_{h\in H} (c_{ijh} + P_{ih}) x_{ijht} \tag{84}$$

subject to

$$\sum_{i\in V} \sum_{h\in H} x_{ijh} = 1 \quad \forall j \in V' \tag{85}$$

$$\sum_{j\in V} x_{ijh} - \sum_{j=1}^{m+n} x_{jih} = 0 \quad \forall h \in H, \forall i \in V \tag{86}$$

$$\sum_{j\in V'} x_{ijh} \leqslant K_{ih} \quad \forall h \in H, \forall i \in D \tag{87}$$

$$\sum_{j \in V} y_{ij} - \sum_{j \in V'} y_{ji} = d_i \quad \forall i \in V' \tag{88}$$

$$y_{ij} \leqslant \sum_{h \in H} Q_h x_{ijh} \quad \forall i \in V', \forall j \in V, \forall h \in H \tag{89}$$

$$x_{ijh} = 0, 1 \quad \forall i, j \in V, \forall h \in H \tag{90}$$

$$y_{ij} \geqslant 0 \quad \forall i, j \in V \tag{91}$$

6.4 INVENTORY ROUTING PROBLEM

When inventory constraints are taken into account in models such as those described in the previous section, we are faced with an IRP. We described a model IRP for one period of time with lost demand. A second model is presented similar to before, but now including several periods into time horizon. The last model included multi-suppliers, multi-vehicles and heterogeneous fleet.

6.4.1 *Basic IRP*

According to [41], in a IRP, both the suppliers and retailers incur unit inventory holding costs h_i per period $(i \in V)$, and each retailer has an inventory holding capacity U_i. The quantity of product made available at the supplier is R_t. We assume that suppliers has enough inventory to meet all the demand and that inventories are not allowed to be negative, i.e., the suppliers can only ship what he holds in stock with no backlogging option. At the beginning of the period the decision maker knows the current inventory level of the suppliers and of the retailers $I_i^0 \forall i \in V$, and receives the information on the demand d of each retailer i for each time period t.

The solution to the problem should determine which retailers to serve using the supply's vehicle and which route to use in one time period. In this special case there is one supplier, so $m = 1$.

$$I_i = I_i^0 + R_i^0 \quad \forall i \in W \tag{92}$$

$$I_i = I_i^0 + q_i - d_i + L_i \quad \forall i \in V' \tag{93}$$

$$\text{minimize} \sum_{i \in V} h_i I_i + \sum_{i \in V'} z_i L_i + \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{94}$$

subject to

$$q_i \leqslant u_i \leqslant Q \quad \forall i \in V' \tag{95}$$

$$0 \leqslant I_i \leqslant U_i \quad \forall i \in V' \tag{96}$$

$$I_i \geqslant \sum_{j \in V'} q_j \quad \forall i \in W \tag{97}$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \tag{98}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V \tag{99}$$

$$U_i \sum_{j \in V'} x_{ji} - I_i \leqslant q_i \leqslant U_i - I_i \quad \forall i \in W \tag{100}$$

$$u_i - u_j + Q(x_{ij}) \leqslant (Q - q_j) \quad \forall i, j \in V' \tag{101}$$

$$q_i \leqslant U_i \sum_{j \in V} x_{ji} \quad \forall i \in V' \tag{102}$$

In the case to want include lost demand it is necessary change the constraint

6.4.2 *Multi-Period IRP*

The solution to the problem should determine which retailers to serve using the supplier's vehicles and which route to use for several time periods in the time horizon. In this special case there is one supplier, so $m = 1$. The constraints used in this model have already explained in previews models. However, all restrictions used in IRP models, again be explained in Model 8.3.1.

$$\text{minimize} \sum_{t \in P} \sum_{i \in V} h_i I_i^t + \sum_{t \in P} \sum_{i \in V'} z_i L_i^t + \sum_{t \in P} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijt} \tag{103}$$

subject to

LEVELS INVENTORY EQUATIONS

$$I_{it} = I_i^{t-1} + R_i^t \quad \forall i \in D, \forall t \in P \tag{104}$$

$$I_{it} = I_i^{t-1} + q_i^t - d_i^t + L_i^t \quad \forall i \in V', \forall t \in P \tag{105}$$

$$L_{it} \geqslant 0 \quad \forall i \in V', \forall t \in P \tag{106}$$

ROUTE CONSTRAINTS

$$\sum_{j \in V} x_{ij}^{t} = 1 \quad \forall t \in P, \forall i \in V', \tag{107}$$

$$\sum_{j \in V} x_{ji}^{t} = 1 \quad \forall t \in P, \forall i \in V', \tag{108}$$

$$\sum_{i \in V} x_{i1t} = 1 \quad \forall t \in P \tag{109}$$

$$\sum_{i \in V} x_{1it} = 1 \quad \forall t \in P \tag{110}$$

VEHICLE CAPACITY CONSTRAINTS

$$\sum_{j \in V} u_{ij}^{t} - \sum_{j \in V} u_{ji}^{t} = q_{i}^{t} \quad \forall i \in V', \forall t \in P \tag{111}$$

$$0 \leqslant u_{ij}^{t} \leqslant Q x_{ij}^{t} \quad \forall i, j \in V, \forall t \in P \tag{112}$$

INVENTORY CONSTRAINTS

$$q_{i}^{t} \leqslant U_{i} - I_{i}^{t} \quad \forall i \in V', \forall t \in P \tag{113}$$

$$q_{i}^{t} \geqslant Ui \sum_{j \in V'} x_{ij}^{t} - I_{i}^{t} \quad \forall i \in V', \forall t \in P \tag{114}$$

$$q_{i}^{t} \leqslant U_{i} \sum_{j \in V} x_{ij}^{t} \quad \forall i \in V', \forall t \in P \tag{115}$$

$$0 \leqslant I_{i}^{t} \leqslant U_{i} \quad i \in V', t \in P \tag{116}$$

$$I_{i}^{t} \geqslant s q_{i}^{t} \in D, t \in P \tag{117}$$

6.4.3 *Multi-Supplier, multi-period and heterogeneous fleet IRP*

We now extended the formulation given by [41]. The problem is defined as a graph $G = (V; A)$ where $V = 1, ..., m+n$ is the vertex set and A is the arc set. The vertex contains the supplier set D, where $D = 1, ..., m$ and the retailers set V' where $V' = m+1, .., m+n$. Both the suppliers and retailers incur unit inventory holding costs h_i per period $(i \in V)$, and each retailer has an inventory holding capacity U_i. The length of the planning horizon is p and, at each time period $t \in T = 1, ..., p$. The quantity of product made available at the supplier is R^t. We assume the suppliers has enough inventory to meet all the demand during the planning horizon and that inventories are not allowed to be negative, i.e., the suppliers can only ship what he holds in stock with no backlogging option. At the beginning of the planning horizon the decision maker knows the current inventory level of the suppliers and retailers $I_i^0 \forall i \in V$, and receives the information about the demand d_i^t of each retailer i for each time period t.

Let be Q_{ih}: vehicle capacity, v: number of vehicles, n: number of retailers, m: number of suppliers and p: number of periods.

$$I_i^t = I_i^{t-1} + R_i^t \quad \forall i \in D, \forall t \in P \tag{118}$$

The inventory level at the suppliers in period t is calculated by Equation 151 and is defined at the beginning of the period and given by its previous inventory level I_i^{t-1}, plus the inventory made available in period R_i^t. The total load shipping in the supplier given by $\sum_{j \in V} u_{ij}^t$, where $i \in D$ and lost demand in the suppliers is not allowed, by this reason $civ[i][t] >= \sum_{j \in D} u_{ij}^t$.

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t + L_i^t \quad \forall i \in V', \forall t \in P \tag{119}$$

Likewise, the inventory level at retailers in period t is calculated by the Equation 152, where the inventory level is updated using its previous inventory level I_i^{t-1}, plus the quantity of product q_i^t shipping in the period t, plus the real demand d_i^t and the lost demand L_i^t.

Let be NV_{ih} the amount of vehicles assigned to supplier i of the type of vehicle h. Then, the amount of vehicles that supplier i has assigned is given for $NH_i = \sum_{h \in H} NV_{ih} \quad \forall i \in D$.

An Integer Programming formulation is used for the problem. In equation 153, the objective function is presented. The objective is the reduction of the total costs considering the hosting inventory, lost demand and transportation costs, consisting this last one, route costs and vehicle costs.

$$\text{minimize} \sum_{t \in P} \sum_{i \in V} h_i I_i^t + \sum_{t \in P} \sum_{i \in V'} z_i L_i^t + \sum_{t \in P} \sum_{i \in V'} \sum_{j \in V} c_{ijh} x_{ijht} + \sum_{t \in P} \sum_{i \in D} \sum_{j \in V'} (c_{ijh} + k_{ih}) x_{ijht} \tag{120}$$

Several constraints are defined for transportation, vehicles and inventories. The first set of Constraints 154 to 157 refer to the supplier's vehicles fleet.

The constraints 154 refers to number of vehicles that can be used by supplier i in each period of time.

subject to

$$\sum_{j \in V'} \sum_{h \in H} x_{ijht} <= NH_i \quad \forall i \in D, t \in P \tag{121}$$

The Constraint 155 refers to that each retailer can be visited by one only vehicle of a same type.

$$\sum_{j \in V'} x_{ijht} <= 1 \quad \forall i \in D, h \in H, t \in P \tag{122}$$

The Constraint 156 is for flow conservation and according to that, the number of vehicles entering one node should be the same that the number of vehicles leaving it.

$$\sum_{j \in V} x_{ijht} = \sum_{j \in V} x_{jjht} \quad \forall i \in D, \forall h \in H, \forall t \in P \tag{123}$$

The Constraint 157 correspond with a sub-tour elimination constraints:

$$\sum_{j \in V} u_{ijt} - \sum_{j \in V'} u_{jit} = q_i^t \quad \forall i \in V', \forall t \in P \tag{124}$$

The Constraints 158 to 160 ensures that the quantities to be delivered to each retail on assigned routes, do not exceed restrictions of capacity of the vehicles, suppliers or retailers.

The constraints related to quantities delivered ensure that the quantity delivered by the supplier's vehicles to each retail i in each period t could fill the retail's inventory capacity if the retail is served, and will be zero otherwise.

$$\sum_{j \in V} u_{ijt} - \sum_{j \in V'} u_{jit} = q_i^t \quad \forall i \in V', \forall t \in P \tag{125}$$

The Constraint 158 ensures that the retail visited receives the amount of product that has been determined.

$$0 <= u_{ijt} <= \sum_{h \in H} Q_h xijht \quad \forall i \in V', \forall j \in V, \forall t \in P \tag{126}$$

In the Constraint 159 ensures that the amount of product being transported in vehicles type h, do not exceed its capacity.

$$I_i^t >= \sum_{j \in V'} u_{jit} \quad \forall i \in S, \forall t \in P \tag{127}$$

The Constraint 160 established the supplier's inventory must be greater than the amount of inventory it delivered in each period.

$$q_i^t <= U_i - I_i^{t-1} \quad \forall i \in V', \forall t \in P \tag{128}$$

$$\sum_{j \in V} x_{ijht} U_i <= q_i^t <= \sum_{j \in V'} x_{ijht} U_i - i_i^t \quad \forall i \in V', \forall h \in K, \forall t \in P \tag{129}$$

The Constraints 161 and 162 ensures that the amount of product to ship to retailers not exceeding its maximum capacity to storage.

$$0 <= i_i^t <= U_i; \quad \forall i \in V, \forall t \in P \tag{130}$$

The Constraints 163 established inventory in the suppliers and retailers must be greater than zero and less than its maximum capacity.

$$x_{ijht} \in 0, 1 \quad \forall i, j \in V, \forall h \in K, \forall t \in P \tag{131}$$

Finally, the constraint 163 and 164 ensures the integrability and non negativity of the variables

6.5 CONCLUSIONS

We formulated the TSP problem as the first problem to address the optimization of logistic process distribution. Besides that, the IRP was presented by means of four versions and identified variables of interest and developed mathematical models. The process of distribution is complemented with the addition of the concept of capacitive vehicles and the inclusion of VRP formulations. We formulated five of the most representative versions of this problem. The integration of the logistic process of inventory was included and tree variants of the IRP problems were formulated. We adapted benchmark instances available in the literature to each formulated model. Therefore, we provided a complete review of the variables and mathematical models used in the TSP, VRP and IRP problems, in order to stablish a foundation for addressing the integrated model for MDSIRP.

Part IV

METAHEURISTIC ALGORITHMS AND EXPERIMENTAL RESULTS

7 INVENTORY REPLENISHMENT AND CUSTOMER SELECTION POLICIES ALGORITHM (IRCSPA)

An article based on this chapter is currently subjected to the final review for publication in Computers & Operations Research by Raúl Roldán, Rosa Basagoiti and Leandro Coelho, which has the title Robustness of inventory replenishment and customer selection policies ([99]).

7.1 INTRODUCTION

Supply chain performance, coordination and integration are some key success factors in obtaining competitive advantages [88]. Inventory and distribution management are two main activities towards that integration, and are said to account for more than 60% of the total logistics costs [51]. The integration of inventory and distribution decisions gives rise to the inventory-routing problem (IRP), which has been studied for the past few decades and has received much attention lately [37]. However, most of these studies focus on optimizing a problem for which all information is known a priori, which is often not the case in practice.

The demand information in an IRP can be static when customers demand are known before the planning, or in a dynamic context in which it is gradually revealed over time [16, 39]. The dynamic and stochastic IRP (DSIRP) aims not at providing a static output, but rather a solution strategy that uses the revealed information, specifying which actions must be taken as time goes by [13]. Recently, Bertazzi et al. [16] and Solyali, Cordeau, and Laporte [111] and Coelho, Cordeau, and Laporte [39] have solved DSIRPs with the goal of minimizing the total inventory, distribution and shortage cost. They considered at least one of the classical inventory policies, i.e., maximum level or order-up-to. They tested their algorithms on instances containing several customers and periods.

An overview of state of the art of IRPs is provided in Roldán, Basagoiti, and Onieva [102] where some key elements were identified that should be taken into account to propose alternative solutions to DSIRPs. The information management between different stakeholders in the supply chain is one of them, since this determines the evolution and quality of shared information. It is necessary to establish inventory management policies, which requires the information sharing between stakeholders. Inventory policies and their relation to the information on the demand is another one, in order to properly manage inventory levels. Finally, one must decide which optimization technique to use in order to make the best use of the available data.

The choice of which inventory policy to apply largely influences the cost of the optimization process. Typically, it uses three key parameters affecting the inventory control: when replenish, how much to replenish, and how often the inventory level is reviewed. For the periodic review inventory system, Wensing [124] describes three policies. One is the order-up-to (OU) which refers to a (t, S) system. Here, in each period t, the quantity delivered is that to fill the inventory capacity up to S. Other policies are the (t, s, S) and the (t, s, q). In the former, the customer is served if the inventory level is less than s. In the latter, the replenishment level q is flexible but bounded by the storage capacity. The policies should be articulated with strategies for clients selection, because sometimes it is not possible to serve all clients due to vehicle capacities, and in such cases, it is necessary to prioritize some of them.

Several other exact and metaheuristic methods have been used to find feasible solutions for this problem and its variants, such as the vehicle routing, where branch-and-cut and evolutionary algorithms are widely used. Simic and Simic [109] argue that for complex optimization problems such as the IRP, hybrid methods with techniques such as artificial neural networks, genetic algorithms, tabu search, simulated annealing and evolutionary algorithms can be successfully applied. Some of the techniques used to solve IRPs are summarized next. Genetic algorithms have been employed by Christiansen et al. [30] and Liu and Lee [79], who clustered customers in geographical areas to serve them together. Local search operators were explored by Javid and Azad [68] and Qin et al. [94], who changed the delivery schedule for customers and adjusted the quantities deliveries accordingly. Li et al. [78] and Liu and Lin [80] and Sajjadi and Cheraghi [105] used simulated annealing to integrate location decisions into the IRP. Adaptive large neighborhood search [38] and a hybrid of mathematical programming and local search [6] have also been used. Finally, exact methods relying on branch-and-cut [8, 40] and branch-cut-and-price [45] have also been developed.

In this paper we study a DSIRP in which decisions must be made without future information about the demand, which is gradually revealed over time. We propose a new three-step solution algorithm, which is flexible enough to consider several inventory replenishment policies. We are then able to evaluate and compare the performance of our policies on demand satisfaction, average inventory kept at the customers' site, transportation cost, and total cost. Moreover, we show the effect of integrating tactical and operational decisions into the same solution algorithm. We compare the performance of our algorithm on benchmark instances available in the literature, and our results show that the right combination of inventory replenishment policies and customer selection can yield significant savings.

The remainder of this paper is organized as follows. In Section 7.2 we formally describe the problem. In Section 7.3 we present our solution procedure which includes customer selection, quantities determination, and vehicle routing. In Section 7.4, we present the results of extensive computational experiments and we analyze the trade-off between inventory and transportation costs. We describe how we can identify dominated solutions under a multi-objective optimization approach, and we compare our solutions against the ones from the literature. In Section 7.5 we present our conclusions and findings.

7.2 PROBLEM DESCRIPTION

The IRP under study consists of one supplier and several retailers as depicted in Figure 18. We assume that the supplier has enough inventory to satisfy the demand of its customers. Customers demand are gradually revealed over time, thus it is said to be dynamic and unknown to the decision maker at the time all decisions are made. The problem is defined over several periods, typically days, and without loss of generality we assume the demand becomes known at the end of the period. This demand can encompass a set of products organized in a pallet, and we will then treat a single commodity as it is done in other IRPs. The supplier has a single capacitated vehicle to distribute the products and to satisfy the final demand of the customers.

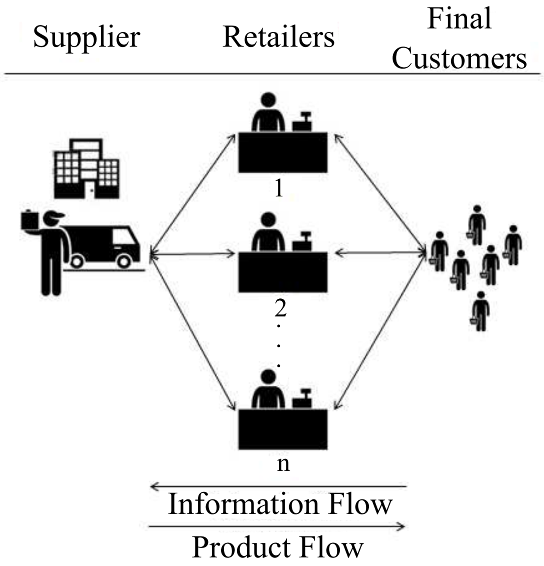


Figure 13: A typical IRP instance with one supplier, n retailers, and a set of final customers representing the demand of the retailers

The IRP is defined with a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \{0, \ldots, n\}$ is the vertex set and $\mathcal{A} = \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ is the arc set. Vertex 0 represents the supplier and the remainder vertices of $\mathcal{V}'$ represent n retailers. The problem is defined over a finite time horizon $\mathcal{H} = \{1, \ldots, p\}$.

The costs incurred are the total of inventory and transportation costs. Inventory costs include the inventory holding and shortage penalties. A transportation cost is paid for each arc traversed by the vehicle. The transportation cost is based on a symmetric distance matrix.

Let n represent the number of customers, each with an initial inventory I_i^0, and let the demand of customer i in period t be d_i^t. Each customer has a maximum inventory capacity C_i, and a unit holding cost h_i is due. Shortages are penalized with z per unit.

A single vehicle with capacity Q is available at the depot. The depot has an initial inventory I_0^0, and inventories incur a unit holding cost h_0. A symmetric transportation cost c_{ij} is known. We denote I_0^t the inventory level at the depot in period t, I_i^t the inventory level at customer i at the end of period t, and l_i^t its lost demand. Let q_i^t be the quantity of product delivered in period t to customer i.

At the end of each period t, the inventory level I_i^t for each customer i is updated based on its demand d_i^t, its lost sales l_i^t, the inventory level at previous period I_i^{t-1}, and the quantity q_i^t delivered to it. A solution to the problem determines the periods in which each customer must be visited, how much to deliver to each of them, and how to create vehicle routes that start at the supplier visit all customers selected to receive a delivery in the period, and return to the supplier location. All capacities must be respected, and stockouts are penalized in order to be avoided.

7.3 SOLUTION ALGORITHM

Our algorithm, Inventory Replenishment and Customer Selection Policies Algorithm (IRCSPA) called, works by decomposing the problem into smaller parts and by solving them using specialized algorithms. The first part of our solution methodology determines which customers to be visited in each period. This can be done in different ways depending on which inventory replenishment strategy is used. We describe the details of this algorithm in Section 7.3.1. The second part of the IRCSPA algorithm determines how much to deliver to each customer in each period. At this phase, the selection of customers is already done, and one must then respect the capacity of the vehicle. The details on how we determine delivery quantities are described in Section 7.3.2. The third and last part of IRCSPA is to create vehicle routes. This problem can be solved by different algorithms. Here, we use a specialized exact algorithm. It is briefly described in Section 7.3.3. A flowchart of our solution method to the problem is depicted in Figure 14.

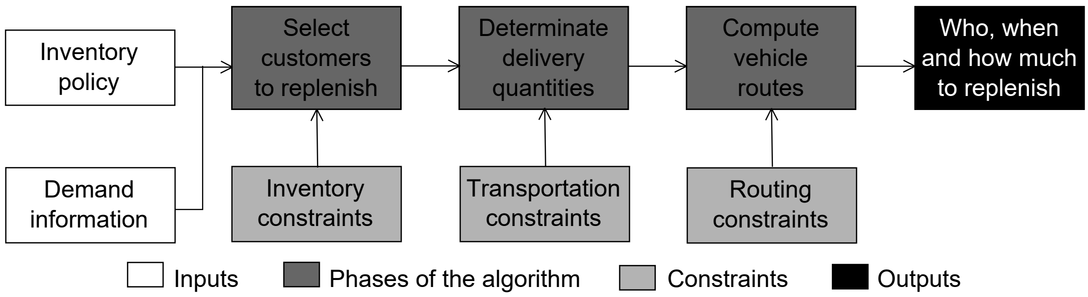


Figure 14: Overview of the main parts of IRCSPA algorithm

7.3.1 *Selecting customers to replenish*

The selection of customers to replenish on a given period depends on the inventory policy used. In what follows we enumerate several different policies organized in four groups in Table 19. They are described next.

1. **Fixed quantity policy**: in this policy, the customer always receives a fixed quantity. The fixed quantities for each customer is defined as a fraction θ of its maximum inventory level, i.e., their inventory capacity. In our experiments, we have set $\theta = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. We note that for $\theta = 0.0$, nothing is shipped and in case of $\theta = 1.0$ this policy coincides with the order-up-to (OU) one. Anything in between yields a maximum level (ML) policy.

2. **OU policy**: the decision maker enforces an OU policy, meaning that whenever a customer is visited, the quantity delivered is that to fill its inventory capacity.

3. **Look ahead**: the decision maker knows a one-step ahead demand. In this case, the delivery quantities are equal to the forthcoming demand.

4. **(s, S)**: the decision maker implements an (s, S) inventory policy. The value of S is set as the inventory capacity, and the parameter s is used to determine when to replenish. This (s, S) policy consists in ordering a variable quantity equal to the difference between S and the current inventory position I_i^t as soon as the inventory level is less than s. The parameter s can be set in some different ways as follows:

   a) The parameter s is determined for each retailer as one fraction α of the inventory capacity, where $\alpha = \{0.25, 0.50, 0.75\}$.

b) The values of the parameter s are computed for each retailer using the mean over its historical data.

c) The value of the parameter s is calculated for each retailer using the mean plus a safety stock, computed as $s = \mu_H + z_\beta \sigma_H$, where β is the probability of a stock-out and z_β is the order quantile of the demand distribution. Here, $1 - \beta$ usually refers to the service level.

d) The value of the parameter s is equal to the one-step ahead demand.

Table 19: Different possible inventory policies

Group	Variant	Policy	Decision
1	$q_{it} = \theta C_i$	ML and OU	ML, if $I_i^t + q_i < C_i^t$ OU, otherwise
2	$q_{it} = C_i^t - I_i^t$	OU	OU, if $C_i^t - I_i > 0$ 0, otherwise
3	$q_{it} = D_i^t$	ML and OU	ML, if $I_i^t + Q_i < C_i^t$ OU, otherwise
4	$s_i = \alpha S_i$ $s_i = \mu_{H_i}$ $s_i = \mu_{H_i} + \sigma_{H_i} z_\beta$ $s_i = d_i^t$	(s, S) (s, S) (s, S) (s, S)	$S_i - I_i^t$, if $I_i^t < s_i$ 0, otherwise

Under policies 1–3 of Table 19, all retailers are set to be visited in every period, and under policies 4 only those whose inventory level is below the reorder point s are selected. In our tests, we have chosen policies 1 and 4a as they are representative of all the possible combinations of parameters and policies.

7.3.2 *Determining delivery quantities*

It is possible that after having selected the customers and an inventory policy, the capacity of the vehicle is not sufficient to guarantee that the policy is fully respected. Different strategies can be applied in order to rectify the situation. In this work, three different strategies are studied.

1. Big Orders First (BOF): under this strategy we prioritize customers requiring more products.

2. Lowest Storage First (LSF): here, we prioritize customers with the low storage capacity.

3. Equal Quantity Discount (EQD): in this strategy, we subtract the same amount to all orders until all customers can be served.

For the first and second strategies, it is important to notice that the last customer selected will be replenished with the remaining capacity of the vehicle.

7.3.3 *Computing vehicle routes*

The remaining step in IRCSPA is to create vehicle routes of minimum distance, leaving the supplier, visiting all selected customers in each period, and returning to the supplier. This problem is an instance of the traveling salesman problem (TSP) [5], a classical combinatorial optimization problem. Solutions for the TSP can be obtained by a myriad of heuristic and exact algorithms. One of these, Concorde [5], is a publicly available algorithm for solving TSPs to optimality. We use this algorithm to obtain solutions for the TSPs arising in our solution method.

At this point, IRCSPA determines the inventory level of each customer, all the incurred costs, and the procedure is repeated for the next period of the planning horizon.

7.4 COMPUTATIONAL EXPERIMENTS

We have implemented IRCSPA algorithm in Matlab 2009b running under Windows 8.1. All computations were performed on a personal computer with Intel Core i3-2370M running at 2.40GHz and with 8GB of RAM memory. We have used the large dataset of instances from Coelho, Cordeau, and Laporte [39]. For early tests we have chosen to use the large instances containing 20 periods, ranging from five to 200 customers, for a total of 10 instances. They are identified as IRP-n-p-i, indicating n customers, and p periods. Each instance was tested under the two proposed inventory policies (with 10 different values for the parameter θ and three values for the parameter α), and for each one of three customer selection strategies.

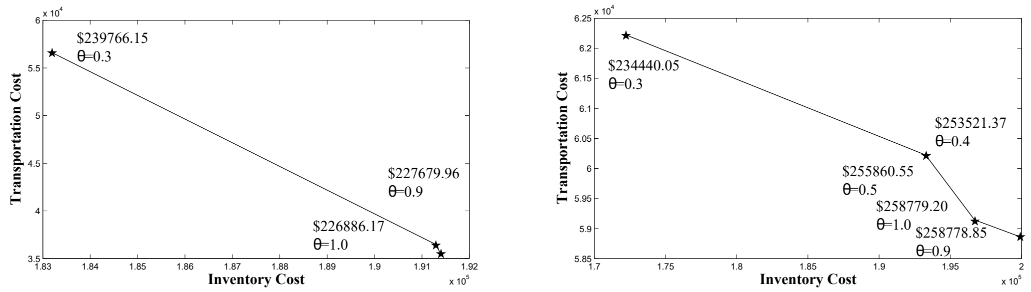
7.4.1 *A multi-objective optimization analysis*

Multi-objective optimization aims at finding Pareto-optimal set or Pareto front consisting of several solutions balancing conflicting objectives. Thus, a multi-objective optimization problem deals with simultaneous optimization of two or more objectives which are contradictory, because improvement in any objective is not possible without degradation in other objectives.

This is a case of the objective of the minimization of transportation cost and minimization of inventory cost. Hence there cannot be a single optimum solution which simultaneously optimizes both objectives. The resulting outcome of a multi-objective optimization is a set of optimal solutions with a varying degree of objective values. This set of solutions is called the non-dominated set or Pareto optimal set. Because minimization of transportation cost and minimization of inventory cost cannot be achieved at the same time, there exists a trade-off between them. This type of system clearly represents a multi-objective optimization situation, in which one looks for a compromise policy, based on a number of options.

We have then solved the instances of the problem using the different methods proposed in this paper. Non-dominated solutions found by the procedure were drawn as points in a plane, with the Y axis representing the transportation cost and the X axis representing the inventory costs. In what follows, each figure depicts the Pareto frontier points with annotations for the total average cost, delivery quantity strategy and inventory policy.

In Figure 15 we show the dominant solutions for the fixed quantity policies under the BOF and LSF delivery strategies. For BOF, three possibilities of replenishment to customers are obtained. The one with $q = 0.3C$ provides a lower inventory cost than those with $q = 0.9C$ and $q = 1.0C$, although the latter yields a lower transportation cost. For the LSF strategy we see five distinct solutions. The EQD policy did not yield different solutions.



(a) BOF policy (b) LSF policy

Figure 15: Pareto frontier for the fixed quantity policy with two different customer selection strategies. The EQD strategy did not yield different solutions.

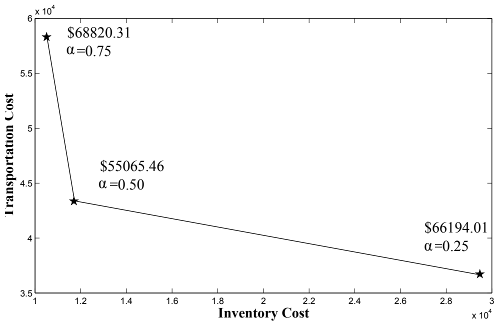
In Figure 20 we show the dominant solutions for the reorder point policies under the BOF, LSF and EQD delivery strategies. For each delivery policy, three distinct and non-dominated solutions were obtained. The lowest transportation cost was always achieved with $\alpha = 0.25$ at the expense of a very high inventory cost. Alternatively, $\alpha = 0.75$ provided the lowest inventory costs, but very high transportation costs.

7.4.2 *Single objective: total cost minimization*

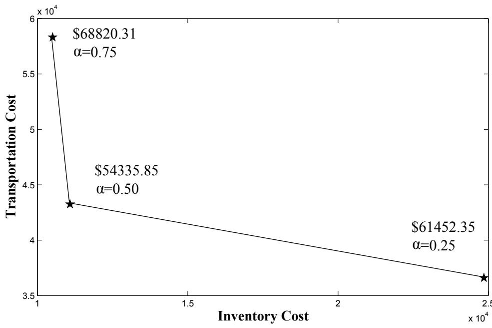
In order to minimize the total cost of inventory and distribution, we have tested the same policies and compared our solutions against those from the literature.

Since this problem allows stockouts, a quick way to find feasible solutions and a benchmark value other than solutions listed in the literature is the case in which the supplier chooses not to replenish, and thus pay the stockout costs. This strategy, also called "wait and see" and coincides with policy number one with θ=0.0. We show its cost in Table 20. The total cost of the system was separated in its inventory, transportation, and stockout components. Obviously, this policy does not perform well and its costs are significantly higher than those of Coelho, Cordeau, and Laporte [39].

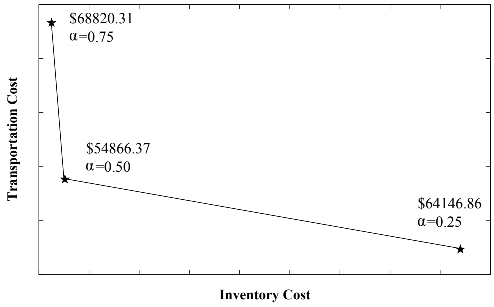
The first of our proposed policies rely on the supplier replenishing each retailer with a predetermined quantity, as computed from policy one from Table 19. Observe that we have evaluated ten different values for the parameter θ. Under these fixed quantity policies, we note that all strategies of delivery quantities presented in Section 7.3.2 (BOF, LSF, and EQD) yielded the same transportation costs due to the vehicle capacity never being exceeded. For this reason, the transportation cost is stable throughout the ten values of θ, whereas stockouts costs are drastically reduced, at the expense of a slight increase on inventory holding costs. The reduction of the average total costs from the $0.5C_i$ to $1.0C_i$ policies are very similar and yield the best comparison against the results of Coelho, Cordeau, and Laporte [39]. The difference in these values arises in the average stockout: while in Coelho, Cordeau, and Laporte



(a) BOF Policy



(b) LSF Policy



(c) EQD Policy

Figure 16: Pareto frontier for the reorder point policy with three different customer selection strategies.

Table 20: Detailed costs for the first policy θ=0.0 compared with those of Coelho, Cordeau, and Laporte [39]

Instance	Inventory holding	Vehicle routing	Stockout	Total cost	Coelho et al. [39]
IRP-5-20	298.37	0.00	45588.00	45886.37	17188.00
IRP-10-20	487.07	0.00	91020.00	91507.07	20182.80
IRP-15-20	842.72	0.00	153808.00	154650.72	33848.20
IRP-25-20	1233.55	0.00	275068.00	276301.55	36455.10
IRP-50-20	2913.80	0.00	506978.00	509891.80	58807.70
IRP-75-20	4025.12	0.00	822502.00	826527.12	77171.90
IRP-100-20	5067.09	0.00	1169380.00	1174447.09	90398.00
IRP-125-20	6870.42	0.00	1349788.00	1356658.42	106242.00
IRP-150-20	7313.46	0.00	1608706.00	1616019.46	114352.00
IRP-200-20	9642.02	0.00	2151808.00	2161450.02	138854.00
Average	3869.36	0.00	817464.60	821333.96	69349.97

[39] there is no stockout, in our policies low values are obtained. Overall, the fixed quantity policy does not outperform the solutions obtained by Coelho, Cordeau, and Laporte [39].

Table 21: Detailed costs for the fixed quantity policy compared those of Coelho, Cordeau, and Laporte [39]. All customer selection strategies yielded the same solution.

Policy	Inventory holding	Vehicle routing	Stockout	Total cost	Coelho et al. [39]
$q = 0.1C_i$	3689.84	63028.93	545634.20	612352.97	69349.97
$q = 0.2C_i$	4616.62	63028.93	275232.80	342878.35	69349.97
$q = 0.3C_i$	8268.17	63028.93	115060.60	186357.69	69349.97
$q = 0.4C_i$	9576.26	63028.93	47539.00	120144.19	69349.97
$q = 0.5C_i$	10712.07	63028.93	1047.80	74788.80	69349.97
$q = 0.6C_i$	10925.41	63028.93	337.20	74291.54	69349.97
$q = 0.7C_i$	10935.15	63028.93	182.80	74146.88	69349.97
$q = 0.8C_i$	10937.22	63028.93	123.00	74089.15	69349.97
$q = 0.9C_i$	10938.00	63028.93	111.60	74078.53	69349.97
$q = 1.0C_i$	10938.34	63028.93	109.20	74076.47	69349.97

The second of our proposed policies is based on replenishments triggered by a reorder point as proposed by item 4a of Table 19. The results obtained are presented in Table 22 for the Big Orders First policy, in Table 23 for the Lowest Storage First policy, and in Table 24 for the Equal Quantity Discount policy. Here, we have tested three different values for the parameter α, and the results show that $\alpha = 0.50S$ yields the best solution cost across all three policies. Moreover, all three policies have outperformed the solutions of Coelho, Cordeau, and Laporte [39], with an average total cost reduced by about 20%.

Table 22: Detailed costs for the reorder point policy under the BOF customer selection strategy, compared to those of Coelho, Cordeau, and Laporte [39]

Policy	Inventory holding	Vehicle routing	Stockout	Total cost	Coelho et al. [39]
$s = 0.25S$	6844.48	36718.53	22631.00	66194.01	69349.97
$s = 0.50S$	8391.94	43360.32	3313.20	55065.46	69349.97
$s = 0.75S$	10371.04	58312.87	136.40	68820.31	69349.97

Having identified that the reorder point policies are the best ones proposed in this paper, we have then applied all its variants, comprising three values of the parameter α and three customer selection strategies, to all instances of the dataset of Coelho, Cordeau, and Laporte [39]. Like those authors, we also report our findings by grouping instances into small, medium and large. These are reported in Table 25 and show that IRCSPA algorithm can always find better solutions than those of Coelho, Cordeau, and Laporte [39]. It also shows that, as previously expected, the policy with $\alpha = 0.50S$ yields the best results. All customer selection methods performed well, but the LSF outperformed the other two by a small margin.

Table 23: Detailed costs for the reorder point policy under the LSF customer selection strategy, compared to those of Coelho, Cordeau, and Laporte [39]

Policy	Inventory holding	Vehicle routing	Stockout	Total cost	Coelho et al. [39]
$s = 0.25S$	6827.28	36601.67	18023.40	61452.35	69349.97
$s = 0.50S$	8388.97	43249.28	2697.60	54335.85	69349.97
$s = 0.75S$	10371.04	5 height8312.87	136.40	68820.31	69349.97

Table 24: Detailed costs for the reorder point policy under the EQD customer selection strategy, compared to those of Coelho, Cordeau, and Laporte [39]

Policy	Inventory holding	Vehicle routing	Stockout	Total cost	Coelho et al. [39]
$s = 0.25S$	6792.95	37341.31	20012.60	64146.85	69349.97
$s = 0.50S$	8406.25	43827.92	2632.20	54866.37	69349.97
$s = 0.75S$	10371.04	58312.87	136.40	68820.31	69349.97

It is relevant to notice that the running times remain low even when the size of the instance increases, unlike the method of Coelho, Cordeau, and Laporte [39]. The difference in our running times between small and large instances is less than one second, which represents approximately doubling the running time, and never achieving two seconds for the large instances. Those of Coelho, Cordeau, and Laporte [39] increase significantly faster, achieving more than 400 seconds. Finally, one can observe that IRCSPA algorithm can better manage the trade-off between stockout costs and overall costs. With respect to the competition, our average lost demand is about four times as high, but the overall cost is significantly decreased.

We have performed sensitivity analyses to identify how the IRCSPA perform and how the solutions change when the distribution capacity is drastically reduced. This experiment is motivated by the fact that for the first policy, the vehicle capacity was not binding. Thus, we have reduced it by 50%. These results are no longer comparable to those of the literature, and a much higher level of lost demand is incurred. The results of these new tests indicate that under the fixed quantity policy, serving big orders first gives significantly better results than prioritizing customers based on their inventory capacities and on decreasing delivery quantities equally among all customers. Moreover, using the reorder point method does not yield better results than the fixed order, despite having some configurations with similar results.

7.5 CONCLUSIONS

In this paper we have solved the Dynamic and Stochastic Inventory-Routing Problem (DSIRP). This problem appears in the literature as that of managing inventory control and distribution simultaneously, minimizing the total inventory holding, transportation, and stockout costs. Customer demands are revealed dynamically over time, thus one must derive a policy to serve customers accordingly. We have proposed several policies and tested different configurations of the fixed quantity and of the reorder point policies. If the vehicle capacity is not sufficient, we have created three

Table 25: Summary of computational results for the reorder point policies under the three customer selection strategies, compared to those of Coelho, Cordeau, and Laporte [39] (150 instances)

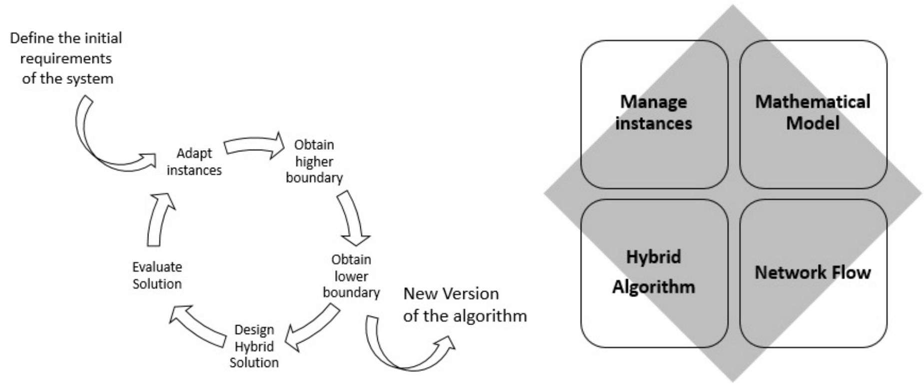
Policy	Instance size	$\alpha = 0.25$			$\alpha = 0.50$			$\alpha = 0.75$			Coelho, Cordeau, and Laporte [39]		
		Cost	Avg lost	Time (s)	Cost	Avg lost	Time (s)	Cost	Avg lost	Time (s)	Cost	Avg lost	Time (s)
BOF	small	**14005.52**	19.70	0.62	15259.22	4.63	0.69	18662.84	0.15	0.70	14974.17	0.62	0.00
	medium	38963.21	15.87	0.81	**36681.73**	3.58	0.88	43137.95	0.06	1.16	39546.01	0.41	4.30
	large	64830.24	15.53	1.01	**56425.41**	3.63	1.22	64539.19	0.06	1.88	64854.75	0.46	408.50
Average		36740.24	17.30	0.79	**34035.83**	4.02	0.91	39768.28	0.10	1.19	39791.64	0.50	137.60
LSF	small	**14085.09**	13.81	0.60	14997.94	1.80	0.70	18796.17	0.07	0.65	14974.17	0.62	0.00
	medium	38819.17	13.11	0.81	**36338.41**	2.13	0.90	43169.84	0.06	1.15	39546.01	0.41	4.30
	large	63197.06	12.83	1.03	**55038.15**	2.15	1.23	64540.46	0.06	1.86	64854.75	0.46	408.50
Average		36238.90	13.30	0.79	**33412.14**	2.00	0.92	39831.55	0.07	1.16	39791.64	0.50	137.60
EQD	small	**15006.36**	15.05	0.61	15370.60	2.02	0.69	18819.59	0.10	0.63	14974.17	0.62	0.00
	medium	40513.46	14.75	0.81	**36573.57**	2.02	0.89	43170.08	0.06	1.14	39546.01	0.41	4.30
	large	66260.57	14.39	1.03	**55457.50**	2.13	1.27	64540.86	0.06	1.87	64854.75	0.46	408.50
Average		38034.75	14.76	0.79	**33757.56**	2.05	0.92	39841.12	0.08	1.16	39791.64	0.50	137.60

strategies to prioritize some customers. We have tested our policies on a large dataset contaning up to 20 periods and 200 customers, and our results significantly improve upon those available in the literature.

8 THE HYBRID GENETIC ALGORITHM WITH NETWORK FLOW FITNESS ALGORITHM (HGANFF)

In vendor-managed inventory, a supplier must determine which customers to visit, how much to replenish, and how to combine them into vehicle routes. This gives rise to the inventory-routing problem. In this paper we analyze a distribution system in which the supplier disposes of several depots and a heterogeneous fleet, and the customers present a dynamic and stochastic demand. In this paper we propose a simple yet effective hybrid genetic algorithm composed of three main components. The first component is a classical genetic algorithm, in which we encode the assignment of customers to depots, obtaining a replenishment schedule pattern. The second component determines vehicle utilization and delivery quantities, and is obtained efficiently in polynomial time for each chromosome. In the final one, demands are realized, total costs consisting of inventory, transportation and lost demand costs are computed, and an acceptance criterion is applied, which corresponds to the fitness value in the genetic algorithm. From a methodological perspective, we propose five new crossover operators and new mutation operators, which have been tested and their performance analyzed on classical benchmark instances. Our method , jointly managing the available stock on many depots, yield an average of 25% improvement over a competing algorithm without transshipment and 18% when the competing algorithm uses transshipment (using the same vehicles).In this chapter a relatively simple but effective hybrid GA to solve the MDDSIRP will be explained and evaluated. The hybrid GA, called HGANFF , was designed following the methodological process explained in Section 5. The incremental parts of the system were: i) manage instances, ii) mathematical modeling, iii) hybrid algorithm and iv) network flow; which are integrated according to the system requirements. In turn, the system follows a iterative process which begins with the definition of the initial requirements of the system. In each iterative cycle, i) one instance is adapted to the available information, ii) the higher and lower boundary to this instances is obtained, iii)An hybrid solution is obtained for it and iv) the solution is evaluated. When a iterative cycle ends, a new version of the hybrid algorithm is created. The combination of the iterative and incremental development allows the solutions to be evaluated properly according to the boundaries found. In terms of reduction of average costs, the results will show a good performance compared with a lower and higher boundaries. In Figure 17 the development methodology used is shown.

The lowers boundaries were provided by the CPLEX implementation and higher boundaries(derived from literature and aggregating costs of individually solved one supplier and many retailers inventory routing problems). Using the solution of a mathematical model for MDDSIRP, a lower limit was established. This model includes an additional relaxation rule, where all suppliers freely could distribute its products to retailers using available vehicles, that is, retailers can be replenished by many suppliers. As for the upper limit, it is set by the results obtained for a single supplier of Coelho heuristic model, but adapted by us to multi suppliers. Starting from three of the Coelho instances, all of them with an equal number of retailers assigned to one unique supplier, are grouped together to formulate a new multi supplier instance



(a) Iterative Development Cycle (b) Incremental Development Parts

Figure 17: Iterative and incremental development methods put together for the HGANFF algorithm

with 3 suppliers that could replenish to any retailer. Therefore, we assume that this is a particular case in which each supplier has been assigned to a group of retailers and then, any accommodation or modification in this assignment should enhance this solution comming from the superposition of instances. For benchmarking, the high boundaries can be calculated as the adding of all the costs of the individual instances used. Regarding to the new hybrid algorithm we propose, one additional constraint is imposed, where each supplier will be assigned to replenish a given group of retailers by each time period. A retailer should be replenished as he demands, or not, depending on the inventory cost, lost demand cost and transportation cost. We assume that it is possible to obtain a better solution than previously mentioned high boundary but hardly enhance the lower boundary. This is due to the additional constraint we establish related to only one supplier replenisihing each retailer.

8.1 INTRODUCTION

In this section we briefly review the existing research on multi-depot IRPs (MDDSIRP) as a complement to Chapter 2. Our focus is on the most relevant elements for the heuristics to solve MDDSIRP problems with many suppliers and heterogeneous vehicle fleet. The objective function will be to minimize the costs of the inventory and transportation of products from many suppliers to many retailers using heterogeneous vehicles. The IRP with multi-depots shows a lack of papers that address the problem in depth its mathematical formulation and implementation. However, we revised several papers that have partially addressed this issue, as well as others that bring different points of view. For instance, a similar procedure to that we will propose, was implementing by [107]. In this paper, a flow-based mathematical formulation and variable neighborhood search implementation for a special case of a VRP that includes Multi-Supplier and Heterogeneous Fleet or Depot and Heterogeneous Fleet Vehicle Routing Problem (MDHFVRP) was proposed. A mathematical formulation was given and lower as well as upper bounds are produced using a three hour execution time of CPLEX. The variable neighborhood search that incorporates new features in addition to the adaptation of several existing neighborhoods and local search operators was proposed. Also, the algorithm was equipped with a scheme for determining borderline retailers, a multi-level heuristic acting as the local search engine, Dijkstra's algorithm for determining the optimal clustering, a diversification procedure and a mechanism to aggregate the routes from different suppliers and disaggregate them into corresponding suppliers accordingly. The difference with our algorithm is that we incorporate inventory constraints and the generic algorithms was used.

In the next paragraphs, we can find more details of the formulation and implementation. The purpose of this paper is to show simple modifications of some well known methods to allow for variable running costs; and also to assess the effect of neglecting such variability. Interesting numerical results, measured in terms of changes in total cost or/and fleet configuration, are found at no extra computational effort.

Considering that the use of clustering methods is important to solve the MDDSIRP problems as can be observed in the literature review presented in Chapter 2, we now mention some of the techniques applied MDDSIRPs. Generally the clustering is decomposed into its two natural components: (1) clustering of edges into feasible routes and (2) actual route construction, with possible feedback loops between the two stages. To bring an overview of the using of these techniques, we trough of the some implementations will describe by means recent articles how the clustering is applied.

For instance, Luo and Chen [83] and Luo and Chen [84] implemented an algorithm that generates clusters randomly to perform the clustering analysis considering the depots as the centroids of the clusters for the retailers. Afterwards, they implemented the local depth search for every cluster, and then, a readjustment of the solutions was performed. In a next step, a new clustering analysis was performed to generate new clusters according to the best solution achieved by the preceding process. The improved path information was inherited to the new clusters, and local search for every cluster was used again iteratively. The process continued until the convergence criteria was satisfied.

Similar process was followed by Zeng, He, and Zheng [131] and He et al. [53], who classified the retailers in certain and uncertain assignment to a supplier, according to the distances of that retailers to suppliers. Their method created an iterative modification of those assignments. When each retailer corresponds to only one supplier, the MDVRP was solved as a single supplier VRP for each supplier in the system.

Xu and Xiao [125], Yücenur and Demirel [130] and Salhi, Thangiah, and Rahman [106] used a technique that allowed implementing a new type of geometric shape based genetic clustering algorithm which could be used effectively to route vehicles if the new shapes have the capability to adapt to the route shapes, resulting in the minimization of the routing cost. The GA is used to adaptively search for the attributes of a set of shapes (example circles) that clusters retailers using the routing cost as the fitness value for the individual chromosomes.

Wang [122] used a typical procedure which consist on the decomposition of a multi suppliers to a single supplier IRP problem. A heuristic was used to simplify the multi-depot problem into a single depot problem. The maximal route is improved with the highest number of retailers and similarly also, by exchanging customers between routes.

Other techniques besides grouping have been used to deal with the MDDSIRP problem. In [117] and [116], a tactical model comprises the models of optimal supply distribution on the battlefield and of optimal reconnaissance by unmanned aerial vehicles used by the military. They used ant colony optimization algorithm with five special forms to select the supplier that should cluster the retailers. These forms include selection of suppliers through a random manner, shortest distance, probabilities according the distance traveled so far and pheromone trails.

Finally, Nananukul [89] illustrated how customers' demands pattern and holding costs could affect their clustering decision. A basic model for clustering customers called multi-period clustering problem was introduced, taking into consideration the demand pattern and holding costs. In this method, an enhanced K-means algorithm was used to construct an initial solution. A novel feature of the algorithm was to create adaptive core clusters which are used in the clustering process instead of the original data points. The neighborhoods of the solution space consist of two types of moves: reassigning customers to clusters and rescheduling the delivery quantity from one period to another. You et al. [126] used clustering and location-based heuristics to group the customers into a number of small clusters and solved the routing problem within each cluster independently. By iteratively changing the customers in the clusters, they obtained a near-optimal solution within the required computational time. The clustering method was integrated into a multi-period two-stage stochastic mixed-integer nonlinear programming model that considered the uncertain demand as random variable.

8.2 MULTI-SUPPLIER, MULTI-PERIOD AND HETEROGENEOUS FLEET IRP

The problem under study contains one supplier with many depots, as depicted in Figure 18, who must satisfy the demand of many customers, and we assume that the supplier has enough inventory to satisfy the demand of the customers. The supplier disposes of a set of heterogeneous vehicles located at each depot. The demand of each customer is gradually revealed over time, thus it is said to be dynamic and unknown to the decision maker at the time all decisions are made. The problem is defined over several periods, typically days, and without loss of generality we assume the demand becomes known at the end of the period. We consider the maximum level (ML) inventory policy, which allows the supplier to freely detertime the quantity to deliver to the customers, limited only by their inventory capacity.

Formally, the MDDSIRP is defined on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \{1, \dots, m, m+1, \dots, m+n\}$ is the vertex set and $\mathcal{A} = \{(i,j) : i,j \in \mathcal{V}, i \neq j\}$ is the arc set. The vertices of $\mathcal{S} = \{1, \dots, m\}$ represents the m depots, and the remainder vertices of $\mathcal{V}' = \mathcal{V} \backslash \mathcal{S}$ represent n customers. The problem is defined over a finite time horizon $\mathcal{P} = \{1, \dots, t\}$.

The costs incurred are the total of inventory and transportation costs. Inventory costs include the inventory holding and shortage penalties. A transportation cost is paid for each arc used by the vehicles and by a fixed vehicle utilization cost. The transportation cost is based on a symmetric distance matrix.

An limited heterogeneous fleet with different capacities is available at each node of $\mathcal{S}$. Let H represent the number of vehicles types, each indexed by h and with capacity Q_h, and let the number vehicles of type h available at each depot $i \in \mathcal{S}$ be N_{hi}. Thus, for convenience, let N_i be the number of vehicles available at depot i, i.e., $N_i = \sum_{h=1}^{H} N_{hi}$. The usage of of vehicle h by depot i incurs a fixed cost k_{hi}.

Each node of $i \in \mathcal{V}$ starts with an initial inventory I_i^0, and the demand of customer $i \in \mathcal{V}'$ in period t is denoted d_i^t, which is not known until the end of period t. Depots receive/produce a quantity r_{it} per period. Each node has a maximum inventory capacity U_i, and a unit holding cost h_i is due. Shortages at the customers are penalized with z_i per unit, but no stockout is allowed at the depots. We denote

I_i^t the inventory of node i in period t, and l_i^t the lost demand of customer i. Let c_{ij} represent a symmetric transportation cost, and v_{ih} a fixed vehicle cost.

At the beginning of each period t, the inventory level I_i^t for each depot $i \in \mathcal{S}$ is updated based on its previous inventory level, on the quantity of products shipped to customers in the previous period, and on the quantity of products r^{it} becoming available. For each customer, the inventory level is updated based on its demand, lost sales, deliveries, and previous inventory level.

A solution to the problem determines the periods in which each customer must be visited, how much to deliver to each of them, and how to create vehicle routes. All capacities must be respected and inventory and transportation costs must be minimized. Since demand is dynamic and stochastic, the output is a policy that determines how the decisions should evolve as a function of the demand in real-time. We consider the inventory policy of maximum level (ML) which allows the supplier to freely choose the quantity to deliver to the customers, limited only by the inventory capacity at the retailers.

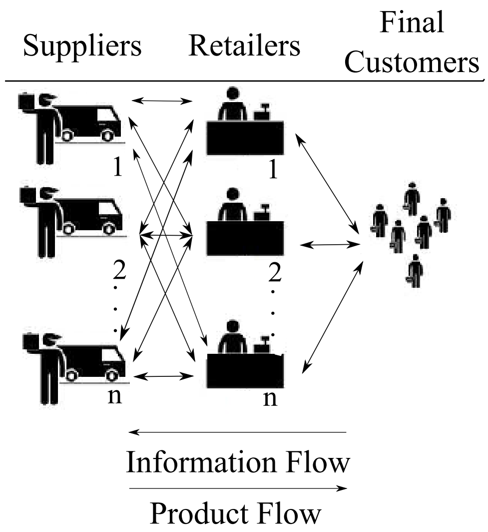


Figure 18: A typical MDDSIRP instance with many suppliers, n retailers, and a set of final customers representing the demand of the retailers

The IRP is defined with a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \{1, \dots, m, \dots, n\}$ is the vertex set and $\mathcal{A} = \{(i,j) : i,j \in \mathcal{V}, i \neq j\}$ is the arc set. Vertex $1, 2 \dots m$ represents the supplier $\mathcal{S}$ and the remainder vertices of $\mathcal{V}'$ represent retailers. The problem is defined over a finite time horizon $\mathcal{P} = \{1, \dots, t\}$.

The costs incurred are the total of inventory and transportation costs. Inventory costs include the inventory holding and shortage penalties. A transportation cost is paid for each arc traversed by the vehicle type. This transportation cost is based on a symmetric distance matrix. There is also a extra fixed cost associated with each additional vehicle used.

A vehicle fleet with different capacities is available per each supplier. Let h represent the number of vehicles types, each with a capacity Q_h, and let the number of one type vehicles available per each supplier i be $NV[i][h]$. Also, consider NH_i as the amount of vehicles that the supplier i has available. It is important to note that each type of vehicle used by one supplier i has an associated cost of $k_i h$.

Let m and n represent the number of suppliers and retailers respectively, each with an initial inventory $I_i^0 \forall i \in V$, and let the demand of customer i in period t be $d_i^t \forall i \in V'$. Each customer has a maximum inventory capacity U_i, and a unit holding cost h_i is due. Shortages are penalized with z per unit.

The suppliers has an initial inventory I_i^0, and inventories incur a unit holding cost $h_i \ \forall i \in S$. A symmetric transportation cost $c_{ij} \ \forall i, j \in V'$ and fixed cost by utilization of the vehicle $v_{ih} \ \forall i \in S and h \in H$ are known . We denote I_i^t the inventory level at the supplier in period $t \ \forall i \in S$, I_i^t the inventory level at Retailer i at the end of period t, and l_i^t its lost demand $\forall i \in V'$. Let q_i^t be the quantity of product delivered in period t to customer i.

At the begin of each period t , the inventory level I_i^t for each supplier $i \in S$ is updated based on quantity of product available R^t and the inventory level at previous period I_i^{t-1}. We assume the suppliers have enough inventory to meet all the demand during the planning horizon and that inventories are not allowed to be negative, i.e., the suppliers can only ship what he holds in stock with no backlogging option. At the end of each period t, the inventory level I_i^t for each customer $i \in V'$ is updated based on its demand d_i^t, its lost sales l_i^t, the inventory level at previous period I_i^{t-1}, and the quantity q_i^t delivered to it.

A solution to the problem determines the periods in which each retailer must be visited, how much to deliver to each of them, and how to create vehicle routes that start at the supplier visit all retailers selected to receive a delivery in the period, and return to the supplier location. All capacities must be respected, and that stockouts are penalized in order to be avoided. In general, the output is a policy that prescribes how the decisions should evolve as a function of the demand in real-time.

8.3 MATHEMATICA MODEL FOR THE MDDSIRP

$$\text{minimize} \sum_{t \in P} \sum_{i \in V} h_i I_i^t + \sum_{t \in P} \sum_{i \in Vc} z_i L_i^t + \sum_{t \in P} \sum_{i \in Vc} \sum_{h \in H} \sum_{d \in Vd} K_{hd} x_{dih}^{dt} + \sum_{t \in P} \sum_{i \in V} \sum_{j \in V} \sum_{h \in H} \sum_{d \in Vd} c_{ij} x_{ijh}^{dt} \tag{132}$$

$$\sum_{j \in V} x_{ijh}^{dt} + \sum_{j \in V} x_{jih}^{dt} = 2y_{ih}^{dt} \quad i \in Vc, h \in H, d \in Vd, t \in P \tag{133}$$

$$\sum_{i \in V} x_{ijh}^{dt} = \sum_{i \in V} x_{jih}^{dt} \quad j \in V, h \in H, d \in Vd, t \in P \tag{134}$$

$$y_{dh}^{dt} \leqslant \sum_{i \in V} \sum_{j \in V} x_{ijh}^{dt} \quad h \in H, d \in Vd, t \in P \tag{135}$$

$$2y_{dh}^{dt} \leqslant \sum_{j \in Vc} x_{jdh}^{dt} + \sum_{j \in Vc} x_{djh}^{dt} \quad h \in H, d \in Vd, t \in P \tag{136}$$

$$x_{ijh}^{dt} = 0 \quad i \in Vc, j \in Vd, j \neq d, h \in H, d \in Vd, t \in P \tag{137}$$

$$x_{ijh}^{dt} = 0 \quad i \in Vd, i \neq d, j \in Vc, h \in H, d \in Vd, t \in P \tag{138}$$

$$\sum_{j \in Vc} \sum_{h \in H} x_{ijh}^{dt} \leqslant nv[i][h] \quad i \in Vd, d \in Vd, t \in P \tag{139}$$

$$\sum_{i \in V} u_{ij}^{t} - \sum_{i \in V} u_{ji}^{t} = q_j^t \quad j \in Vc, t \in P \tag{140}$$

$$\sum_{i \in Vd} \sum_{j \in Vc} u_{ij}^{t} = \sum_{j \in Vc} q_j^t \quad t \in P \tag{141}$$

$$u_{ij}^{t} \leqslant \sum_{h \in H} \sum_{d \in Vd} (Q_h - q_i) x_{ijh}^{dt} \quad i \in V, j \in Vc, t \in P \tag{142}$$

$$0 \leqslant x_{ijh}^{dt} \leqslant 1 \quad i, j \in V, h \in H, d \in Vd, t \in P \tag{143}$$

$$0 \leqslant y_{ih}^{dt} \leqslant 1 \quad i \in V, h \in H, d \in Vd, t \in P \tag{144}$$

$$I_i^t \geqslant \sum_{j \in Vc} u_{ji}^{t} \quad i \in Vd, t \in P \tag{145}$$

$$q_i^t \leqslant (U_i - I_i^{t-1}) \quad i \in Vc, t \in P \tag{146}$$

$$\sum_{j \in V} x_{ijh}^{dt} \leqslant q_i^t \tag{147}$$

$$q_i^t \leqslant \sum_{j \in Vc} U_i x_{ijh}^{dt} - I_i^t \quad i \in Vc, h \in H, t \in P \tag{148}$$

$$0 \leqslant I_i^t \quad j \in V, t \in P \tag{149}$$

$$I_i^t \leqslant U_i \quad i \in V, t \in P \tag{150}$$

8.3.1 *Multi-Supplier, multi-period and heterogeneous fleet IRP*

We now extended the formulation given by [41]. The problem is defined as a graph $G = (V; A)$ where $V = 1, ..., m + n$ is the vertex set and A is the arc set. The vertex contains the supplier set D, where $D = 1, ..., m$ and the retailers set V' where $V' = m + 1, .., m + n$. Both the suppliers and retailers incur unit inventory holding costs h_i per period $(i \in V)$, and each retailer has an inventory holding capacity U_i. The length of the planning horizon is p and, at each time period $t \in T = 1, ..., p$. The quantity of product made available at the supplier is R^t. We assume the suppliers has enough inventory to meet all the demand during the planning horizon and that inventories are not allowed to be negative, i.e., the suppliers can only ship what he holds in stock with no backlogging option. At the beginning of the planning horizon the decision maker knows the current inventory level of the suppliers and retailers $I_i^0 \forall i \in V$, and receives the information about the demand d_i^t of each retailer i for each time period t.

Let be Q_{ih}: vehicle capacity, ν: number of vehicles, n: number of retailers, m: number of suppliers and p: number of periods.

$$I_i^t = I_i^{t-1} + R_i^t \quad \forall i \in D, \forall t \in P \tag{151}$$

The inventory level at the suppliers in period t is calculated by Equation 151 and is defined at the beginning of the period and given by its previous inventory level I_i^{t-1}, plus the inventory made available in period R_i^t. The total load shipping in the supplier given by $\sum_{j \in V} u_{ij}^t$, where $i \in D$ and lost demand in the suppliers is not allowed, by this reason $civ[i][t] >= \sum_{j \in D} u_{ij}^t$.

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t + L_i^t \quad \forall i \in V', \forall t \in P \tag{152}$$

Likewise, the inventory level at retailers in period t is calculated by the Equation 152, where the inventory level is updated using its previous inventory level I_i^{t-1}, plus the quantity of product q_i^t shipping in the period t, plus the real demand d_i^t and the lost demand L_i^t.

Let be NV_{ih} the amount of vehicles assigned to supplier i of the type of vehicle h. Then, the amount of vehicles that supplier i has assigned is given for $NH_i = \sum_{h \in H} NV_{ih} \quad \forall i \in D$.

An Integer Programming formulation is used for the problem. In equation 153, the objective function is presented. The objective is the reduction of the total costs considering the hosting inventory, lost demand and transportation costs, consisting this last one, route costs and vehicle costs.

$$\text{minimize} \sum_{t \in P} \sum_{i \in V} h_i I_i^t + \sum_{t \in P} \sum_{i \in V'} z_i L_i^t + \sum_{t \in P} \sum_{i \in V'} \sum_{j \in V} c_{ijh} x_{ijht} + \sum_{t \in P} \sum_{i \in D} \sum_{j \in V'} (c_{ijh} + k_{ih}) x_{ijht} \tag{153}$$

Several constraints are defined for transportation, vehicles and inventories. The first set of Constraints 154 to 157 refer to the supplier's vehicles fleet.

The constraints 154 refers to number of vehicles that can be used by supplier i in each period of time.

subject to

$$\sum_{j \in V'} \sum_{h \in H} x_{ijht} <= NH_i \quad \forall i \in D, t \in P \tag{154}$$

The Constraint 155 refers to that each retailer can be visited by one only vehicle of a same type.

$$\sum_{j \in V'} x_{ijht} <= 1 \quad \forall i \in D, h \in H, t \in P \tag{155}$$

The Constraint 156 is for flow conservation and according to that, the number of vehicles entering one node should be the same that the number of vehicles leaving it.

$$\sum_{j \in V} x_{ijht} = \sum_{j \in V} x_{jjht} \quad \forall i \in D, \forall h \in H, \forall t \in P \tag{156}$$

The Constraint 157 correspond with a sub-tour elimination constraints:

$$\sum_{j \in V} u_{ijt} - \sum_{j \in V'} u_{jit} = q_i^t \quad \forall i \in V', \forall t \in P \tag{157}$$

The Constraints 158 to 160 ensures that the quantities to be delivered to each retail on assigned routes, do not exceed restrictions of capacity of the vehicles, suppliers or retailers.

The constraints related to quantities delivered ensure that the quantity delivered by the supplier's vehicles to each retail i in each period t could fill the retail's inventory capacity if the retail is served, and will be zero otherwise.

$$\sum_{j \in V} u_{ijt} - \sum_{j \in V'} u_{jit} = q_i^t \quad \forall i \in V', \forall t \in P \tag{158}$$

The Constraint 158 ensures that the retail visited receives the amount of product that has been determined.

$$0 <= u_{ijt} <= \sum_{h \in H} Q_h xijht \quad \forall i \in V', \forall j \in V, \forall t \in P \tag{159}$$

In the Constraint 159 ensures that the amount of product being transported in vehicles type h, do not exceed its capacity.

$$I_i^t >= \sum_{j \in V'} u_{jit} \quad \forall i \in S, \forall t \in P \tag{160}$$

The Constraint 160 established the supplier's inventory must be greater than the amount of inventory it delivered in each period.

$$q_i^t <= U_i - I_i^{t-1} \quad \forall i \in V', \forall t \in P \tag{161}$$

$$\sum_{j \in V} x_{ijht} U_i <= q_i^t <= \sum_{j \in V'} x_{ijht} U_i - i_i^t \quad \forall i \in V', \forall h \in K, \forall t \in P \tag{162}$$

The Constraints 161 and 162 ensures that the amount of product to ship to retailers not exceeding its maximum capacity to storage.

$$0 <= i_i^t <= U_i; \quad \forall i \in V, \forall t \in P \tag{163}$$

The Constraints 163 established inventory in the suppliers and retailers must be greater than zero and less than its maximum capacity.

$$x_{ijht} \in 0, 1 \quad \forall i, j \in V, \forall h \in K, \forall t \in P \tag{164}$$

Finally, the constraint 163 and 164 ensures the integrability and non negativity of the variables

8.4 SOLUTION ALGORITHM

In this section we present the hybrid GA we propose for the solution of the MDDSIRP. Our algorithm is based on the framework of the hybrid GA of Vidal et al. [120] for the multi-depot and periodic VRP. However, our algorithm includes the exact solution of a network flow and of a traveling salesman problem as part of the evaluation of each chromosome. New crossover operators are also proposed. This type of algorithm is highly suitable for the problem at hand because of its generality and flexibility. It can simultaneously handle several families of hard constraints and it conducts a highly diversified search through the multiplicity of its operators.

A general view of our hybrid GA is presented in Algorithm 1. It starts by generating a random population, in which each individual represents a schedule pattern of service. This pattern is used to determine which customers to visit in each time period, including the information about which depot to use. The schedule pattern is then used to instanciate a network flow, whose optimal solution determines the best way to distribute the products from depots to customers in each period. Inventory costs are obtained from this solution. Routing costs are computed solving a TSP instance for each vehicle used. The total of the inventory and transportation costs constitute the fitness value. Finally, the population is updated through genetic operators such as elitism, crossover and mutation, and a new generation is obtained. The process is iterated until a stopping criteria is satisfied.

We now describe each feature of our algorithm, which is composed of four main components. The main genetic framework is described in Section 8.4.1; the network flow problem used to compute vehicle utilization and inventory costs is presented in Section 8.4.2; the routing aspect is detailed in Section 8.4.3; and the acceptance and stopping criteria is presented in Section 8.4.4.

8.4.1 *Genetic algorithm*

In our implementation, each chromosome represents a replenishment scheduling pattern in which customers are assigned to depots in each day. The representation

Algorithm 1 General procedure of our hybrid genetic algorithm

Create a randomly generated population to represent scheduling patterns
while Stopping criteria is not met **do**
  for each new individual **do**
    Determine vehicle utilization, inventory and demand satisfaction by means of a network flow problem
    Determine transportation costs as the sum of the solutions of TSPs for each vehicle and period
    Evaluate the fitness of each individual as sum of inventory and transportation costs
  end for
  Evolve the current population by applying selection, crossover, mutation and elitism operators on its individuals
end while
return the best individual of the population

of a chromosome is described in Section 8.4.1.1. We then create an initial population by creating a set of randomly chosen chromosomes, as detailed in Section 8.4.1.2. Genetic operators such as selection, crossover and mutation are discussed in Section 8.4.1.3. An stopping criteria is also defined and presented in Section 8.4.4.

8.4.1.1 *Representation of the chromosomes*

In Figure 19, two feasible forms to group retailers are showed. The links among these nodes do not represents the order at the time to be served, these only represent the group of retailers that belong to the same group. Inspired by the work of Vidal et al. [120], the individuals representing them in our implementation are represented by a set of two chromosomes: the first one, called the Customer Schedules Chromosome (CSC), encodes which depot serves the customer in each period, if any; the second, called the Giant Tour Chromosome (GTC), contains for each combination (Supplier, Period), a sequence of customers without trip delimiters, obtained by concatenating all routes from each depot for each period. We illustrate these two chromosomes in Figure 20, in which two feasible solutions are represented: in Part (a) the CSC chromosome, and in Part (b) the GTC chromosome.

Regarding the search space, the number of possible combinations is $(m+1)^{(n*H)} - 1$. For instance, the search space in a problem with 3 depots, 45 customers and 5 periods is 2.91×10^{135}. It is important to note that because lost sales are allowed in our problem, all possibilities that can be generated for the RSC and GTC chromosomes are feasible.

8.4.1.2 *Initial population*

We generate a set of initial chromosomes by randomly assigning customers to depots and to periods.

8.4.1.3 *Genetic operators*

The operators we have designed for our algorithms are described next.

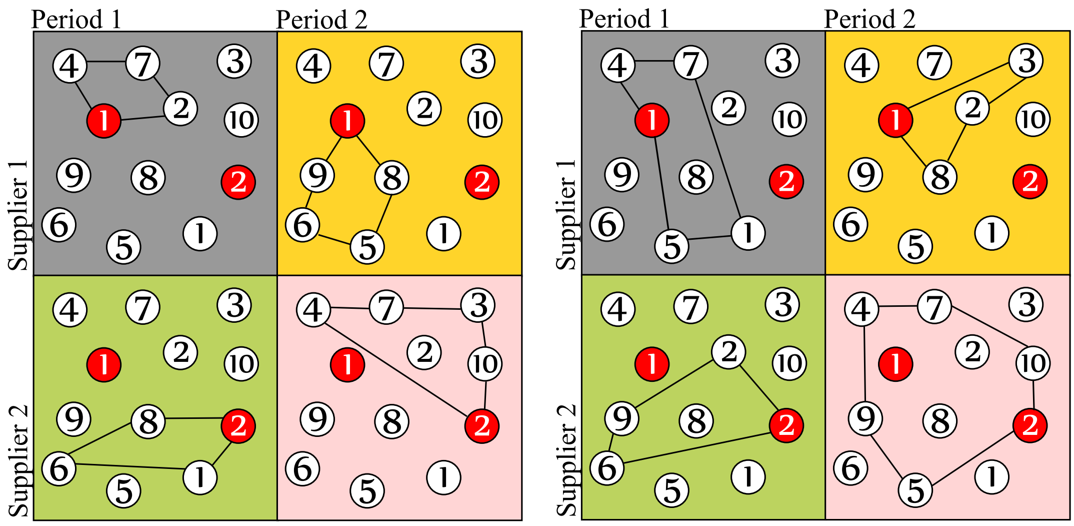


Figure 19: MDDSIRP feasible scheduling for a instance of two depots, ten customers and two periods

Retailer	1	2	3	4	5	6	7	8	9	10
Scheduling by Period	2	1	0	1	0	2	1	2	0	0
	0	0	2	2	1	1	2	1	1	2

Retailer	1	2	3	4	5	6	7	8	9	10
Scheduling by Period	1	2	0	1	1	2	1	0	2	0
	0	1	1	2	2	0	2	1	2	2

(a) Scheduling chromosome for customer replenishment

Giant tour 1	2	4	7	5	6	8	9	1	6	8	3	4	7	10	
Giant tour 2	1	4	5	7	2	3	8	2	6	9	4	5	7	9	10

(b) Routing chromosomes

Figure 20: Chromosome representation of one feasible scheduling solution

SELECTION A selection operator chooses chromosomes from the current population for reproduction. In our algorithm, parent selection is performed through a binary tournament. Two different chromosomes are selected from the population and the chromosome with better fitness is chosen. The procedure is performed twice to select the two parent individuals, namely P_1 and P_2. Selection is done with replacement, which means that the same chromosome can be selected more than once for reproduction.

CROSSOVER Crossover is the main genetic operator and consists of swapping chromosome parts between parents P_1 and P_2 . Crossover is not performed on every pair of selected individuals, and its frequency is controlled by a crossover probability. We have developed five crossover operators. These operators are described next and the general procedure to implement the crossover operator is shown in Algorithm 2.

Algorithm 2 Crossover operations of our algorithm

```
while population is completed do
    Select two parent solutions P1 and P2 by binary tournament
    if crossover probability if accepted then
        Generate offspring C1 and C2 by crossover operations
        Insert C1 and C2 into the population
    else
        Insert P1 and P2 into the population
    end if
end while
```

Five different operators were developed. The first one was based in the work of Vidal et al. [120]. We used the periodic crossover with insertions (PIX) operator dedicated to periodic vehicle routing problems and adapted it for the IRP. PIX crossovers use two giant tours and the basic procedure is represented in Figure 21.

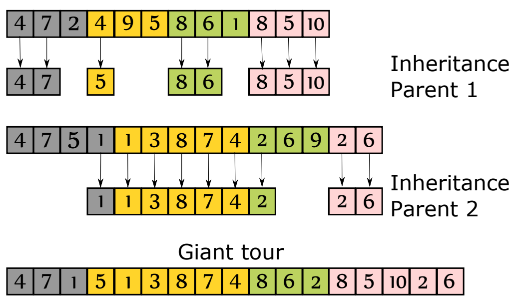


Figure 21: Procedure to generate an offspring by PIX crossover

The second and third operators are 2-point and 3-point crossover, respectively (denoted as CX2 and CX3). The CSC is used to produce an offspring with heritage patterns regarding periods and depot assignment. The basic procedure is represented in the Figure 22.

The fourth crossover method takes into account ideas from the convex set theory and the work of Kaelo and Ali [69]. In the case of the CSC each value is a number between 0 to m and an arithmetic crossover (AMX) is used. Simple arithmetic opera-

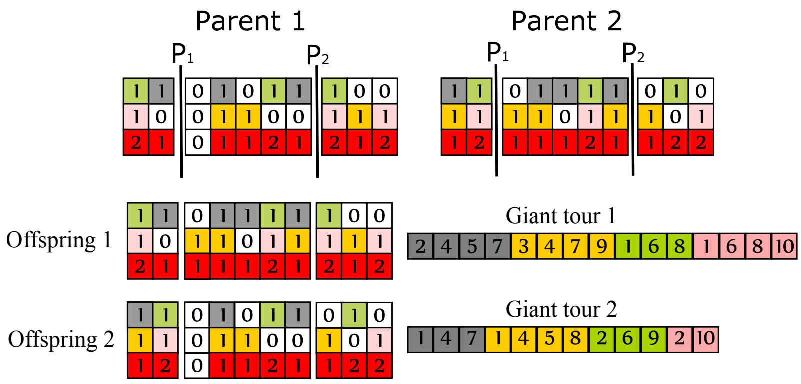


Figure 22: Procedure to generate two offspring by CX2 and CX3 operators

tors are defined as the convex combination of two vectors (chromosomes) according to Equation 165:

$$C_1 = \alpha P_1 + (1 - \alpha) P_2. \tag{165}$$

Where the α is an uniform random number r/m equal, where r is a random integer number in the range of 0 and m. The application of a linear combination in this operator with a parameter $\alpha \in [0, 1]$ as a method of gene recombination guarantees closeness of this operator. This procedure can be used twice to generate two offspring.

The last operator developed is an average crossover (AVX) that takes two parents and returns one offspring as described by Equation 166:

$$C_1 = \frac{1}{2}(P_1 + P_2). \tag{166}$$

MUTATION Several mutation operators are used to avoid the algorithm being stuck in local optima. These are described next.

1. Flip-Addition (FA): this is a gene mutation operator. The genes of the offspring are changed according to a probability. The change consists of increasing its current value in one. If the gene has the value of m (the maximum allowed), its value is changed to zero.

2. Flip-Random (FR): likewise, this operator changes genes of the offspring. The change is to assign a value different from the current gene value. The value must lie between $[0, m]$.

3. Scramble-Depot: This is a permutation operator that changes the current depot for each customer in a random way. The genes of the offspring are changed according to a probability.

4. Insertion: The insertion mutation operator selects a gene randomly and inserts it at a random position. This procedure, that is a permutation operator, produces a displacement of the genes located between the two positions interchanged.

5. Power Mutation (PM): The power mutation operator is based on a power distribution. Its distribution is shown in Equation 167.

$$f(x) = \rho x^{\rho-1} 0 \leqslant x \leqslant 1 \tag{167}$$

$$PM = \begin{cases} \text{if } t < \alpha & x - \gamma(x - l) \\ \text{Otherwise} & x + \gamma(u - x) \end{cases} \tag{168}$$

Where γ follows the power distribution, $t = \frac{x-l}{u-l}$, l and u are lower and upper bounds, in this case $l = 0$ and $u = m$. Let be α be a number draw from a uniform distributed between 0 and 1. The strength of power mutation is governed by the index of the mutation ρ. For small values of ρ less disturbance in the solution is expected. For large values of ρ more diversity is achieved. The probability of producing a mutated solution on left (right) side of the C_i is proportional to distance of x from l (u) and the new mutated solution always remains feasible.

8.4.2 *The minimum-cost flow problem*

All information regarding scheduling patterns is passed to a network flow solver to simultaneously optimize inventory, delivery quantities, lost demand and vehicles to be used. A minimum cost network flow is computed in order to obtain the best way to distribute the products from depots to customers for each period. This is done following the developments proposed in Coelho, Cordeau, and Laporte [38, 39].

8.4.3 *Routing*

Once the CSC determines which vehicles from each depot to assign to each customer, and the network flow has computed delivery quantities that respect vehicle capacities, one must route customers in vehicle routes starting and finishing at the depots. The problem can be decomposed by depot, by vehicle and by period. Thus, the solution is equivalent to the solution of a TSP, one for each combination of vehicle, depot and period. We solve several TSPs and compute the total transportation cost of the chromosome.

8.4.4 *Stopping criteria*

In our implementation, we have limited the number of generations to 100, and the maximum number of iterations without improvement to 10.

8.5 COMPUTATIONAL RESULTS

Our (HGANFF) algorithm was coded in C++ using Microsoft Visual Studio 2012. We used the algorithm for the minimum-cost flow problem to solve the second level problem(once the genetic algorithm decided the suppliers attending each retailer). It was run on an Intel Core 5-4210M 2.60GHz and 8 GB RAM laptop PC. The mathematical model was coded in C++ using IBM Concert Technology and CPLEX 12.6.1 with four threads. Also at this level, Concorde Algorithm was used to calculate the near-optimal routes for the retailers assigned to each vehicle. Computations were executed in a laptop PC with Windows operating system.

To evaluate the performance of our algorithms, we have used the instances of the IRP generated and solved to near-optimality by [42]. These instances are for one single supplier and were adapted for the MDDSIRP problem. Three of the smallest instances, those that used 5, 10 and 15 retailers (each instance has 5 different versions), were grouped together in order to formulate a new multi supplier instance with 3 suppliers and 15, 30 and 45 retailers respectively (There were 6 forms to group them) and the details about their generation can be consulted in Appendix D. For benchmarking, the overall high boundaries for the costs of the created new instances can be calculated as the adding of all the costs of the individual instances because, we assume that this is a particular solution in which each supplier has a group of retailers previously assigned.

In the Coelho, Cordeau, and Laporte [42] instances, a vehicle with a given capacity was used to perform the routes. For the new instances, we tested the utilization of 2 types of vehicles of different capacity, where each supplier could use one vehicle of each type. To establish the capacity for each type of vehicle, we sum the independent capacity of the each vehicle of the Coelho instance and according to this data, chosen and taken the 40% of it for the first vehicle type and 60% for the second one. Considering that this value is a key in the construction of the routes, we have decided to divide the instances in those with high, medium and low vehicle capacity and test the algorithm for all of them. For this reason, 9 cases are reported, 3 for each capacity (15, 30 and 45 retailers).

In Coelho, Cordeau, and Laporte [42] 2 types of the solutions are presented. In the first one, the total costs are calculated as a reaction to the previous day demand. In the second one, transshipment is allowed . As mentioned before, these values are taken as the hight boundary for the comparison of our results.

There are no previously reported solutions for the MDDSIRP since we are introducing the problem in this paper. For this reason, we have compared our HGANFF algorithm against the optimal solutions obtained with the mathematical model described in the section 8.2 and used this results as a lower boundary. Two executions were performed, in the first one, CPLEX compiler was run for 4500 seconds as time limit, therefore the problem of the system going out memory was avoided. For the second one, in order to avoid big losses due to the lost demand, the lost demand cost was penalized, multiplying it by 10.

In Tables 26 to 28 we reported the heuristic solutions present by Coelho, Cordeau, and Laporte [42] and denoted as Coelho RS (reactive standard), and Coelho TRS (transshipment and reactive). The solutions obtained with the HGANFF algorithm and its variants were denoted as GA PIX, GA CX2, GA CX3, GA AMX and GA AMX-AVX (as well as the version for $10 * \text{losses}$). Finally, the optimal solutions obtained by the mathematical model for the two executions mentioned (CPLEX 4500 sec and CPLEX $10 * \text{losses}$) were also added to the table.

Regarding the content of the columns of these tables, 10 columns were used to present type of technique used to solve the problem, number of generations used by the algorithm, inventory hosting cost of the solution, the lost demand cost, transportation costs and overall costs (as the sum of inventory losses and transportation), mean of the lost demand cost by period, as well as the percentage of gap respect to the lower bounds found by the CPLEX implementation.

The Results for HGANFF for high capacity vehicles are reported in Table 26, medium capacity in Table 27 and low capacity in Table 28. In each table the average of three instances tested are showed such as 15, 30 and 45 retailers, all with 3 suppliers. In accordance with the instances that were generated, each supplier has two vehicles available, one with higher capacity than the other. However, in the case of high vehicles capacity, the suppliers only have the need to use one type of them to face the retailers demand on each period of the time. As the capacity of the vehicles decreases for the other two cases, the suppliers begin to use both types of vehicles and consequently, the total costs are higher. For this reason, we decided to test three cases of vehicles capacity. It is important to note that in the Coelho instances solutions, we assume that three vehicles were used, meanwhile, in our solutions, to be 3 three supplier and 2 vehicles type, with one only vehicle type by retailer, the amount of vehicles can reach to 6.

The results show that in all the three cases (high, medium and low vehicles capacity), we have enhanced the overall costs in the solutions, with respect of RS and TRS reactive solutions of Coelho, Cordeau, and Laporte [42] with the $GA-AMX-AVX$ algorithm variant (only in one case with $GACX_2$). Also, the average lost demand cost was improved in special for the $10*\mathrm{losses}$ solutions, tested for all GA algorithms. Using the $GA-AMX-AVX$ algorithm, improvements in both figures were achieved.

Regarding the computation time, the solutions obtained with $GA-AMX-AVX$ get the alternative solution in less time than the optimal solution calculated by CPLEX, when original losses are implemented as $10*\mathrm{losses}$. The computation times of Coelho instances can not be compared, because it is not possible superimpose the computation times.

Out of the 9 instances tested, our HANDOFF algorithm was able to match the solution values on 100% of instances, also, it improved the solution values near to 10% of the cases of the reactive form and these solutions showed an competitive performance with respect the optimal values given by CPLEX.

8.6 CONCLUSIONS

We have implement an algorithm to solve the multi-supplier and multi-vehicle Dynamic-Inventory-Routing Problem, were the retailers are clustered by suppliers and each supplier has one fleet of vehicles of different type available. This problem is very difficult to solve exactly but it is possible to generate good enough solutions in a limited time horizon. Comparative tests on a large set of artificial instances have shown that our algorithm can produce high quality solutions within reasonable computing times.

Table 26: Results for HGANFF for high capacity vehicles compared to Coelho RS and TRS as higher and CPLEX solution as lower boundaries respectively

mddsirp-3-15/3-30/ 3-45	High vehicle capacity								
Technique used	**Gen**	**Inv Cost**	**losses Cost**	**Trans Cost**	**Total Costs**	**AvLost Cost**	**Time sec**	**gap(%)**	**gap(%) (10*losses)**
Coelho RS	–	–	–	–	15890.39	37.63	–	43.72	30.09
Coelho TRS	–	–	–	–	14609.72	0.00	–	38.45	23.78
GA PIX	43	1391.67	402.00	12683.33	14477.00	80.40	5199.67	36.69	–
GA CX1	28	1402.00	504.67	13798.67	15705.33	100.93	3323.33	42.71	–
GA CX2	28	1406.33	398.00	14372.67	16177.00	79.60	3302.67	44.51	–
GA AMX	47	1361.33	344.00	11482.00	13187.33	68.80	5061.00	30.45	–
GA AMX-AVX	43	1337.33	846.00	8966.33	**11149.67**	169.20	3194.00	19.60	–
GA PIX (10*LOSSES)	39	1438.72	10.67	16640.00	18089.38	2.13	4183.00	–	38.95
GA CX1 (10*LOSSES)	28	1460.99	10.67	16645.00	18116.65	2.13	2807.00	–	38.52
GA CX2 (10*LOSSES)	37	1440.85	4.67	15557.67	17003.19	0.93	3768.67	–	34.23
GA AMX (10*LOSSES)	59	1413.27	8.67	14218.67	15640.60	1.73	6190.00	–	28.67
GA AMX-AVX (10*LOSSES)	33	1396.22	76.00	10421.33	**11893.56**	15.20	2373.00	–	6.26
CPLEX (4500sec)	–	1226.51	598.67	7336.90	9162.08	119.73	3216.33	0.00	–
CPLEX (10*LOSSES)	–	1298.14	23.33	10257.82	11579.29	4.67	4500.67	–	0.00

Table 27: Results for HGANFF for medium capacity vehicles compared to Coelho RS and TRS as higher and CPLEX solution as lower boundaries respectively

mddsirp-3-15/3-30/ 3-45	Medium vehicle capacity								
Technique used	**Gen**	**Inv Cost**	**losses Cost**	**Trans Cost**	**Total Costs**	**AvLost Cost**	**Time sec**	**gap(%)**	**gap(%) (10*losses)**
Coelho RS	–	–	–	–	15890.39	37.63	–	35.71	32.59
Coelho TRS	–	–	–	–	14609.72	0.00	–	29.88	26.29
GA PIX	27	1399.33	184.00	14616.67	16200.00	36.80	4287.67	34.60	–
GA CX1	27	1427.00	439.33	15800.67	17667.00	87.87	3604.33	42.34	–
GA CX2	44	1412.67	322.67	13941.33	15676.67	64.53	5566.67	30.60	–
GA AMX	48	1384.33	175.33	13877.33	15437.00	35.07	6396.67	34.23	–
GA AMX-AVX	48	1281.00	1700.00	10611.33	**13592.33**	340.00	4175.33	25.53	–
GA PIX (10*LOSSES)	56	1453.20	16.00	17204.33	18673.53	3.20	5751.67	–	42.35
GA CX1 (10*LOSSES)	35	1442.63	39.33	17379.67	18861.63	7.87	3593.33	–	43.54
GA CX2 (10*LOSSES)	36	1463.22	8.00	16652.33	18123.56	1.60	3640.67	–	40.85
GA AMX (10*LOSSES)	47	1423.71	8.67	15173.00	16605.38	1.73	5205.33	–	35.37
GA AMX-AVX (10*LOSSES)	30	1352.05	82.67	12846.00	**14280.72**	16.53	2547.33	–	25.54
CPLEX (4500sec)	–	1262.57	489.33	8861.08	10612.98	97.87	3196.33	0.00	–
CPLEX (10*LOSSES)	–	1288.01	16.00	9667.61	10971.62	3.20	4500.33	–	0.00

Table 28: Results for HGANFF for low capacity vehicles compared to Coelho RS and TRS as higher and CPLEX solution as lower boundaries respectively

mddsirp-3-15/3-30/ 3-45	Low vehicle capacity								
Technique used	**Gen**	**Inv Cost**	**losses Cost**	**Trans Cost**	**Total Costs**	**AvLost Cost**	**Time sec**	**gap(%)**	**gap(%) (10*losses)**
Coelho RS	–	–	–	–	15890,39	37,63	–	29,27	4,73
Coelho TRS	–	–	–	–	14609.72	0.00	–	22.52	-3.81
GA PIX	53	1277.33	518.00	15335.33	17130.67	103.60	7302.67	30.65	–
GA CX1	33	1331.00	1316.67	16929.00	19576.67	263.33	4789.67	40.96	–
GA CX2	36	1334.00	950.00	16951.00	19235.00	190.00	5143.00	40.09	–
GA AMX	48	1333.33	942.00	15964.33	18239.67	188.40	8128.33	37.72	–
GA AMX-AVX	51	1251.33	1628.00	11891.00	**14770.33**	325.60	4906.00	25.14	–
GA PIX (10*LOSSES)	49	1400.12	17.33	21121.00	22538.45	3.47	6636.00	–	31.75
GA CX1 (10*LOSSES)	38	1451.51	14.67	22669.00	24135.18	2.93	5580.00	–	36.28
GA CX2 (10*LOSSES)	44	1425.74	28.67	21048.00	22502.41	5.73	6503.33	–	31.80
GA AMX (10*LOSSES)	54	1392.98	10.67	18640.00	20043.65	2.13	8020.33	–	22.07
GA AMX-AVX (10*LOSSES)	42	1301.15	89.33	14503.00	**15893.48**	17.87	3845.67	–	5.10
CPLEX (4500sec)	–	1247.90	458.00	9625.68	11331.58	91.60	4500.67	0.00	–
CPLEX (10*LOSSES)	–	1357.69	13.33	14468.33	15839.35	2.67	4500.67	–	0.00

9 CONCLUSIONS

In this thesis we have introduced, modeled and solved several types of inventory-routing problems. In particular, we have identified opportunities for new research in single and multi-supplier IRPs, increased flexibility and robustness within these problems and developed hybrid algorithms for their solution. In the next paragraphs, we outline our main findings as well as suggestions for future research.

We have proposed a comprehensive literature review in Chapters 2 and 3. Through the review of papers dealing with stochastic demand and lead time, focusing on its stochastic and multi-supplier aspects, we have identified critical factors for the performance of many logistic activities and industries. Also, we have shown that studying the behavior of the demand and the lead time is essential to achieve a meaningful representation of the system to make proper decisions. By detecting the lack of scientometrics studies, we proposed a study of this type for the IRP. We used the key elements identified to design a search equation for extracting and collecting information from Scopus and Web of Science databases. Our study showed trends and patterns by means of tables in different topics of interest for each research. This, in turn, has enabled us to understand the state of the research in the area of the IRP, specifically for its stochastic, dynamic and under periodic revision of the inventory versions. Moreover, we identified prolific researchers and research groups in the most representative fields of study in IRP.

We have presented in Chapter 4 our research methodology. By the gaps identified in the literature, future work and research questions, we propose a general way to address future research of IRPs. Also, we identified areas of impact which the researchers with interest might further develop. We established the most important operational variables to address the IRP research and evaluated heuristic and exact techniques. In Chapter 5, a consistent methodology for this development was presented. This methodology is based on iterative and incremental developments. The unified process was divided into four phases and tasks that could be integrated. The first phase was to define the problem and the integration was an identification of variables under study. For the second one, the use of instances was required and the mathematical model was used as an integrated task. In the third phase, the design of an algorithm was proposed where each version could be used to integrate it within the development. Finally, for the last phase, an evaluation of the solutions was carried out. The integration of all the phases of development allows us to propose several models to coordinate IRP systems.

In Chapter 6, we formulated the TSP as being the first problem in which we need to address in order to optimize product distribution. Bearing this in mind, the IRP was presented by means of four versions and we identified variables of interest and developed mathematical models. The process of distribution is complemented with the addition of the concept of capacitated vehicles and the inclusion of VRP formulations. We formulated six of the most representative versions of this problem. The integration of the logistic process of inventory management was included and three variants of the IRP were formulated. We adapted benchmark instances available in the literature for each formulated model. Therefore, we provided a complete review of the variables and mathematical models used in the TSP, VRP and IRPs, in order to provide a foundation for addressing the integrated model for MDDSIRP.

We have introduced robustness of inventory replenishment and retailer selection policies in Chapter 7. Our analysis was conducted on the single-supplier case. New retailer selection methods for a dynamic and stochastic inventory-routing problem was proposed. We have developed an algorithm, called IRCSPA, which works by decomposing the problem into smaller parts and by solving them using specialized algorithms. The first part of our solution methodology was to determine which customers to visit in each period; the second part of the solution algorithm determined how much to deliver to each customer in each period; the last part was to create vehicle routes. A multi-criteria analysis of the costs associated with the solutions, comparing distribution versus inventory management was performed. Also, a single criteria objective experiment was performed, showing that our methods yield an average of 20% improvement over a competing algorithm.

Finally in Chapter 8, we used mathematical models and heuristic algorithms to design and implement a hybrid algorithm called HGANFF (Hybrid Genetic Algorithm Netwok Flow) 2to solve the MDDSIRP problem. The algorithm framework proposed for HGANFF was made up of four main components. The first component was a genetic algorithm, which in each iteration, suppliers are assigned to many retailers for each period of time obtaining the replenishment scheduling pattern. For the second component, the information of the scheduling pattern was passed to a network flow solver to optimize the delivery quantities and vehicles to be used. In the third component, the retailers were replenished using using a fleet available at each supplier. In the final one, an acceptance and stopping criteria was applied to obtain the solution as the sum of inventory cost, lost demand and transportation cost. We used this value as the fitness value in the genetic algorithm. Regarding the genetic algorithm, we proposed five new crossover operators, which have been tested and their performance analyzed. Likewise, we propose new mutation operators to use in combination with other operators in the HGANFF. For the analysis of the results, a single criteria objective experiment was performed. Also, we proposed new benchmarks instances to test MDDSIRPs. Our methods, jointly managing the available inventory on many depots, yield an average of 25% improvement over a competing algorithm without transshipment and 18% when the competing algorithm uses transshipment (using the same vehicles).

There are two possible extensions to our work. These are related to flexibility and consistency of the solutions. As flexibility we refer to allow direct shipments and allow movements of products among retailers, as long as the retailer faces stock-outs. Often flexibility is achieved by using outsourced carriers and it is defined in the terms of their contracts which are distance- and volume-dependent. Although the flexibility is a good alternative for eliminating the stock outs, this increases the total costs. It is important to notice that our algorithms has achieved solutions with very low losses without using transshipment. Further tests will show their power at the time of finding solutions without stock-outs and costs associated with these solutions.

The other extension is related to consistency, to add quality of the service to the solutions. In our work, consistency appears in an indirect manner at the time of doing several tests with many different inventory policies and service levels in the Chapter 7. However, these characteristics can be used directly in the IRP through the use of fleet size and managing vehicle load, as well as analyzing the frequency of the deliveries, and to quantities delivered which have not been tested enough. Furthermore, in Chapter 8, we begin to introduce consistency to test solutions with different vehicle capacities. We believe that we can still make many contributions in this direction.

The IRP was introduced approximately 30 years ago and has since evolved into a rich research area. We believe this thesis has helped fill some gaps in this body of knowledge and will stimulate other researchers to pursue the study of this fascinating field.

Part V

APPENDIX

A

DATASET FOR TSP FORMULATIONS

The instances processed by Coelho in [36] were used and adapted, if needed, for testing all the models. Also, new instances by combining them were created. In some cases the Coelho instances were used partially and in others, some data was added. All cases are explained in the following paragraphs where instances initially created for one supplier and several retailers step by step will be modified. For a better understanding, we have included an example with a small instance.

The instances explained here was used for testing the lineal programming mathematical models in Section 6.2 which were solved with CPLEX in version 12.6.1.

In this appendix there are 4 type of instances:

- TSP Basic
- mTSP, Multi-Travelling TSP
- Multi-supplier TSP
- Multi-supplier and Multi Tour TSP

These instances correspond to the ones used in each TSP model formulated in Section 6.2.

In a TSP basic instance, the first line contains the data of retailers number n. In the second line, the supplier identification number idS and its location in Cartesian coordinates xS and yS are given. From third line to the last and until to complete the data for all the retailers, the identification number of retailer idR and its location xR and yR are given. Below, an example for a instance of 1 supplier and 10 retailers is presented:

```
10
1 90   184
1 249  483
2 470  415
3 136  385
4 143  124
5 334  89
6 168  359
7 265  313
8 271  265
9 149  381
10   356  378
```

In mTSP, the information about the number of vehicles that perform the routes was needed. Similar to the previous instances, the first line is used to indicate that the instance is for n retailers, but adding the number of vehicles (VN), as the previous example, the second line brings the identification number and coordinates of suppliers. Next lines, show the identification number and coordinates of the retailers. Below an example for an instance of 1 supplier and 10 retailers and two vehicles is presented:

```
10   2
1  90   184
1  249  483
2  470  415
3  136  385
4  143  124
5  334  89
6  168  359
7  265  313
8  271  265
9  149  381
10   356  378
```

Regarding Multi-supplier TSP, the number of suppliers that replenish retailers and the number retailers is required. In this case, the first line is used to indicate that the instance is for m supplier and n retailers, as the previous instance, the second line has the identification numbers and coordinates of suppliers and next lines shows the identification numbers and coordinates of the retailers. Below, an example for an instance of 2 suppliers and 10 retailers is presented:

```
2  10
1  90   184
2  110  200
1  249  483  98
2  470  415  35
3  136  385  60
4  143  124  65
5  334  89   41
6  168  359  83
7  265  313  12
8  271  265  94
9  149  381  75
10   356  378  44
```

The last type instance, the Multi-supplier and Multi Tour TSP, uses three data in the first line such as suppliers numbers, retailers numbers and vehicles numbers. As there are two suppliers, the lines two and three are used to define their coordinates, the number of vehicles of each type and the fixed cost by vehicle type. For the remaining lines, the coordinates, and demand for each retailer are defined. Below, an example for an instance of 2 suppliers, 10 retailers and 2 vehicles is presented:

```
2  10  2
1  90   184
2  110  200
1  249  483  98
2  470  415  35
3  136  385  60
4  143  124  65
5  334  89   41
6  168  359  83
7  265  313  12
8  271  265  94
9  149  381  75
10   356  378  44
```

B DATASET FOR VRP FORMULATIONS

The instances processed by Coelho in [36] were used and adapted, if needed, for testing all the models. Also, new instances by combining them were created. In some cases the Coelho instances were used partially and in others, some data was added. All cases are explained in the following paragraphs where instances initially created for one supplier and several retailers step by step will be modified. For a better understanding, we have included an example with a small instance.

The instances explained here were used for testing the lineal programming mathematical models in Section 6.3 which were solved with CPLEX in version 12.6.1.

In this appendix there are 4 type of instances:

- CVRP homogeneous fleet
- CVRP heterogeneous fleet
- Multi supplier CVRP homogeneous fleet
- Multi-supplier CVRP heterogeneous fleet
- Multi-supplier CVRPTW heterogeneous fleet

These type of instances correspond to the ones used in each VRP model that was formulated in Section 6.3

In a CVRP homogeneous fleet type instance, the first line contains the data of the retailers amount n and the vehicle capacity Q. Also, in this line, it is possible to set the amount of vehicles to use indicated with the parameter K. In the second line, the supplier identification number idS and its location in Cartesian coordinates xS and yS are given. From the third line to the last and until to complete the data for all retailers, the identification number of retailer idR and its location xR and yR are given. Below an example for an instance of 10 retailers, and 1 vehicle is presented:

```
10   855
1  409  104
2  160  499
1  299  144
2  175  140
3  137  216
4  47   455
5  104  82
6  246  264
7  25   145
8  341  169
9  450  449
10   208  334
```

In a CVRP heterogeneous fleet type instance, the first line contains the data retailers amount n, the vehicle capacity Q_h by type and the fixed cost ch_{mh} corresponding to each vehicle type h. In second line, the identification supplier number idS and its location in Cartesian coordinates xS and yS are given. From the third line to the last and until to complete the data for all retailers, the identification number of retailer idR and its location xR and yR are given. Below, an example for an instance of 10 retailers, and 2 vehicles type is presented:

```
10  393 462 0.3 0.5
1 409 104
1 299 144
2 175 140
3 137 216
4 47  455
5 104 82
6 246 264
7 25  145
8 341 169
9 450 449
10  208 334
```

In a Multi supplier CVRP homogeneous fleet, the first line contains the number of suppliers, retailers amount n and the vehicle capacity Q. Also, in this line, it is possible to agree the amount of vehicle to use indicated with the parameter K. In second line, the supplier identification number idS and its location in Cartesian coordinates xS and yS are given. From third line to the last and until to complete the data for all retailers, the identification number of retailer idR and its location xR and yR are given. Below, an example for an instance of 2 suppliers, 10 retailers and 1 vehicle type is presented:

```
2 10 855
1 409 104
2 160 499
1 299 144
2 175 140
3 137 216
4 47  455
5 104 82
6 246 264
7 25  145
8 341 169
9 450 449
10  208 334
```

In a Multi supplier CVRP heterogeneous fleet, the first line contains the number of suppliers, retailers amount n, the vehicle capacity Q_h by type and the fixed cost ch_{mh} corresponding to each vehicle type h. In the second line, the identification supplier number idS and its location in Cartesian coordinates xS and yS are given. From third line to the last and until to complete the data for all retailers, the identification number of retailer idR and its location xR and yR are given. Below, an example for an instance of 2 suppliers, 10 retailers and 2 vehicles type is presented:

```
2 10  393 462 0.3 0.5
1 409 104
2 160 499
1 299 144
2 175 140
3 137 216
4 47  455
5 104 82
6 246 264
7 25  145
8 341 169
9 450 449
10  208 334
```

In the Multi-supplier CVRPTW heterogeneous fleet instance type, it is necessary to add the date for the time window and the service time. In the first line the number of retailers, the vehicle capacity Q_h by type and the fixed cost ch_{mh} corresponding to each vehicle type h is set. Only one supplier is considered. The second line defines the coordinates of the supplier, time windows and service time. In the remaining lines the coordinates, demand, time windows and service time for each retailer is defined. Below, an example for an instance of 2 suppliers, 10 retailers, 2 vehicles type, time windows and service time is presented:

```
2 10   393 462    0.3 0.5
1 409 104 0 300 0
2 160 499
1 299 144
2 175 140
3 137 216
4 47  455
5 104 82
6 246 264
7 25  145
8 341 169
9 450 449
10  208 334
```

C

DATASET FOR IRP FORMULATIONS

The instances used to test the models are described in this section.

The instances processed by Coelho in [36] were used and adapted, if needed, for testing all the models. Also, new instances by combining them were created. In some cases the Coelho instances were used partially and in others, some data was added. All cases are explained in the following paragraphs where instances initially created for one supplier and several retailers step by step will be modified. For a better understanding, we have included an example with a small instance.

For Section 6.4.1, the start inventory data and the inventory capacity are taken into account.

In this appendix there are 3 type of instances:

- IRP basic formulation
- Multi Period IRP
- Multi supplier and Multi period IRP heterogeneous fleet

In IRP basic formulation, the first line contains the number of retailers and vehicle capacity. In the second line, the information of the supplier is defined as follows: coordinates, starting inventory, available inventory and host inventory cost. For the retailers, coordinates, starting inventory, demand, inventory capacity, host inventory cost and penalty cost for lost demand are defined. Below, an example for an instance of 10 retailers and 1 vehicle is presented:

```
10  934
0 90   184 1810  1369  0.01
1 249 483 200 98   300 0.04  8
2 470 415 31  35   62  0.09  18
3 136 385 61  60   122 0.05  10
4 143 124 210 65   280 0.08  16
5 334 89  51  41   102 0.09  18
6 168 359 76  83   152 0.07  14
7 265 313 42  12   56  0.08  16
8 271 265 178 94   267 0.08  16
9 149 381 228 75   304 0.06  12
10  356 378 110 44  165 0.02  4
```

For Multi Period IRP additional data for each time period is added. In the first line, the number of retailers, number of periods and vehicle capacity are defined. In the second line, coordinates, starting inventory, available inventory for each period and host inventory cost are defined. In the remaining lines, data for each retailer is defined: coordinates, starting inventory, demand for each period, inventory capacity, host inventory cost and penalty by lost demand. Below, an example for an instance of 10 retailers, 5 periods of time and 1 vehicle is presented:

```
10  5 934
0 90   184 1810  1369  1372  1371  1371  1368  0.01
1 249 483 200 98   103 97   106 94   300 0.04  8
2 470 415 31  35   17  36   34  34   62  0.09  18
3 136 385 61  60   64  56   60  60   122 0.05  10
```

```
4 143 124 210 65 63 75 75 73 280 0.08 16
5 334 89 51 41 54 47 52 42 102 0.09 18
6 168 359 76 83 68 72 77 81 152 0.07 14
7 265 313 42 12 16 0 5 3 56 0.08 16
8 271 265 178 94 92 85 86 90 267 0.08 16
9 149 381 228 75 75 76 73 80 304 0.06 12
10 356 378 110 44 61 52 57 54 165 0.02 4
```

For the last type, Multi supplier and Multi period IRP with heterogeneous fleet, it is necessary to add data such as vehicle capacity, number of vehicles by supplier and type and cost by vehicle utilization. The description of the instances is as following: the first line has the number of retailers, number of suppliers, number of vehicle types, number of periods and vehicles capacity. The next two lines defines the data for the supplier as follows: the coordinates, starting inventory, available inventory for each period, host inventory cost, number of vehicles by type and utilization cost for vehicle type. In the remaining lines, the coordinates, starting inventory, demand by period of time, inventory capacity, host inventory cost and lost demand cost per unit are established for each retailer. Below, an example for an instance of 2 suppliers, 10 retailers, 3 vehicles type and 5 periods of time is presented:

```
2 10 3 5 200 300 434
1 90 184 910 669 572 871 871 568 0.01 2 3 4 0,1 0,2 0,3
2 150 200 900 700 800 500 500 800 0.02 1 2 1 0,2 0,4 0,6
1 249 483 200 98 103 97 106 94 300 0.04 8
2 470 415 31 35 17 36 34 34 62 0.09 18
3 136 385 61 60 64 56 60 60 122 0.05 10
4 143 124 210 65 63 75 75 73 280 0.08 16
5 334 89 51 41 54 47 52 42 102 0.09 18
6 168 359 76 83 68 72 77 81 152 0.07 14
7 265 313 42 12 16 0 5 3 56 0.08 16
8 271 265 178 94 92 85 86 90 267 0.08 16
9 149 381 228 75 75 76 73 80 304 0.06 12
10 356 378 110 44 61 52 57 54 165 0.02 4
```

D

DATASET FOR MDDSIRP FORMULATIONS

The instances used to test this type of models are described in this section.

The instances processed by Coelho in [36] were used and adapted, if needed, for testing all the models. Also, new instances by combining them were created. In some cases the Coelho instances were used partially and in others, some data was added. All cases are explained in the following paragraphs where instances initially created for one supplier and several retailers step by step will be modified. For a better understanding, we have included an example with a small instance.

These instances are for one single supplier and were adapted for the MDDSIRP problem. Three of these smallest instances, those 5, 10 and 15 retailers (each instance with 5 different versions), were grouped in order to formulate a new multi supplier instance with 3 suppliers and 15, 30 and 45 retailers respectively (There are 6 forms to group it). For benchmarking, the overall high boundaries for the costs of the created new instance can be calculated as the adding of all costs of the individual instances used. We assumed that this is a particular case in which each supplier has previously assigned a group of retailers.

For theses instances, it is necessary to add data such as vehicle capacity, number of vehicles by supplier and type and cost by vehicle utilization by supplier. The description of the instances is as following: in the first line has the number of retails, number of supplier, number of vehicle types, number of periods and vehicles capacity. The next two lines defines the data for the supplier as follows: the coordinates, starting inventory, available inventory for each period, host inventory cost, number of vehicles by type and utilization cost for vehicle type. In the remaining lines, the coordinates, starting inventory, demand by period of time, inventory capacity, host inventory cost and lost demand cost per unit are established for each retailer. Below, an example for an instance of 3 suppliers, 15 retailers, 2 vehicles type and 5 periods of time is presented:

```
3 15  2 5 502 753
1 409 104 830 671 667 667 673 668 0.01  1 1 0,5 0,6
2 160 499 1.031 657 647 652 651 659 0.01  1 1 0,5 0,6
3 316 252 726 644 643 644 643 646 0.01  1 1 0,5 0,6
1 299 144 132 58  48  41  48  55  176 0.04  8
2 175 140 178 86  92  80  91  82  267 0.07  14
3 137 216 74  32  37  42  30  45  111 0.06  12
4 47  455 76  35  42  37  33  30  114 0.08  16
5 104 82  108 50  53  41  59  50  162 0.07  14
6 246 264 150 81  79  85  62  70  225 0.08  16
7 25  145 104 46  53  44  42  49  156 0.06  12
8 341 169 37  33  37  40  36  37  74  0.03  6
9 450 449 162 53  60  50  59  53  216 0.08  16
10  208 334 270 92  89  92  86  90  360 0.07  14
11  403 237 201 66  70  67  65  70  268 0.08  16
12  395 36  36  18  0 10  8 6 48  0.03  6
13  250 117 83  74  74  83  83  100 166 0.07  14
14  182 357 68  28  36  33  31  34  102 0.05  10
15  496 202 71  72  74  65  73  67  142 0.07  14
```

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