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Novel SHM method to locate damages in substructures based on VARX models

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Abstract. A novel damage localization method is proposed, which is based on a substructuring approach and makes use of Vector Auto-Regressive with eXogenous input (VARX) models. The substructuring approach aims to divide the monitored structure into several multi-DOF isolated substructures. Later, each individual substructure is modelled as a VARX model, and the health of each substructure is determined analyzing the variation of the VARX model. The method allows to detect whether the isolated substructure is damaged, and besides allows to locate and quantify the damage within the substructure. It is not necessary to have a theoretical model of the structure and only the measured displacement data is required to estimate the isolated substructure's VARX model. The proposed method is validated by simulations of a two-dimensional lattice structure.

1. Introduction

Structural Health Monitoring (SHM) is the process of implementing a damage detection and characterization strategy for engineering structures [1]. SHM is regarded as a very important engineering field in order to secure structural and operational safety; issuing early warnings on damage or deterioration, avoiding costly repairs or even catastrophic collapses [2].

Most of the existing vibration based SHM methods could be classified into two different approaches: global approaches and local approaches [3]. In the global approaches, the goal is to monitor the health of the entire structure. These global methods have been tested and implemented in different types of structures during the last 30 years [4]. However, for many large systems, global monitoring is not practical due to the lack of sensitivity of global features regarding local damages, inaccuracies of developed models or the high cost of sensing, cabling and computational operations [5]. On the other hand, local SHM methods are focused on evaluating the state of reduced parts within the entire structures, based on substructuring methods. This approach aims to overcome global method's problems, dividing the whole structure into substructures and analyzing each one individually.

Several research works have proposed substructuring methods for large-scale structures. Koh [6] presented a “divide and conquer” strategy to monitor large structures based on the division of the whole structure into isolated substructures. For each substructure, structural parameters are identified



using the Extended Kalman filter (EKF). However, the EKF usually require knowledge of the system and its dynamics [7]. Yun and Lee [8] detected damage in structures combining a substructuring method and experimental modelling [9]. Most recently, Xing [10] presented another damage detection method based on a substructuring method and Auto-Regressive Moving Average with eXogenous input (ARMAX) models. Damage indicators were obtained for each estimated substructural model calculating the difference between the squared natural frequencies in the healthy state and the squared natural frequencies during the structure lifetime. All the natural frequencies were computed from their respective estimated ARMAX models. This damage detection method was validated through simulations and experimental test. The method proposed by Xing [10] doesn't require any theoretical model of the structure [9]. Nevertheless, as only one internal DOF is measured in each substructure, is not possible to give information about the damage location within the substructures.

In this paper, a damage localization method based on the combination of a substructuring method and experimental modelling is proposed. The substructuring method is used to isolate a multi-DOF substructure from the rest of the structure, and each isolated substructure is modelled as a Vector Auto-Regressive with eXogenous input (VARX) model. VARX models incorporate data measured in different internal DOFs and their coefficient matrices describe the relationship between the measured internal DOFs through some structural characteristics (mass, stiffness, damping...). Therefore, the proposed method could potentially locate the damage within the substructure by analyzing variations on the VARX model over the time. Furthermore, the proposed method doesn't require any theoretical model of the structure.

The rest of the paper is organized as follows. First, the proposed method is presented in section 2. Secondly, the proposed method is evaluated by series of simulations. In section 3, simulation results are discussed and finally the concluding remarks are presented in section 4.

2. The proposed method

The behavior of the structure is described by a lumped parameter model, where we assume that all objects are rigid bodies and all interactions between the rigid bodies take place via springs and dampers. We assume the structure consists of bars that are connected together by rigid joints. The point in which two or more bars are joined is called node and the number of structural nodes will depend on the topology of the structure. The forces could only be transmitted along the axial direction of the bars and the load could only be applied at the two ends of each bar. We assume that the mass of each bar is distributed equally between its two nodes, a half in the first one and the other half in the second one. On the other hand, the structure could be subjected to arbitrary external loading that is assumed to be known and could act on any node.

The structure is divided into different substructures and these substructures are isolated from the remaining structure. The substructures could contain several internal (i) and interface (j) nodes. The interface nodes are located in the border between the selected substructure and the remaining structure. On the other hand, internal nodes are located within the substructure and they are not connected to the nodes of the remaining structure.

The dynamic equations for an internal node i are formulated as follows:

$$\begin{aligned} m_i \ddot{\mathbf{z}}_{xi} &= \sum_{k=1}^N (f_{xk}(\mathbf{z}_i, \dot{\mathbf{z}}_i, \mathbf{z}_k, \dot{\mathbf{z}}_k)) + \mathbf{F}_{xi} \\ m_i \ddot{\mathbf{z}}_{yi} &= \sum_{k=1}^N (f_{yk}(\mathbf{z}_i, \dot{\mathbf{z}}_i, \mathbf{z}_k, \dot{\mathbf{z}}_k)) + \mathbf{F}_{yi} \\ m_i \ddot{\mathbf{z}}_{zi} &= \sum_{k=1}^N (f_{zk}(\mathbf{z}_i, \dot{\mathbf{z}}_i, \mathbf{z}_k, \dot{\mathbf{z}}_k)) + \mathbf{F}_{zi} \end{aligned} \quad (1)$$

where m_i is the lumped mass of the internal node i and $\ddot{\mathbf{z}}_{xi}$, $\ddot{\mathbf{z}}_{yi}$ and $\ddot{\mathbf{z}}_{zi}$ are the absolute accelerations of the internal node i in x, y and z axes respectively. f_{xk} , f_{yk} and f_{zk} are linear or non-linear functions used to calculate the total internal force applied in the node i by the N nodes connected to him. These functions depend on the value of $\mathbf{z}_i, \dot{\mathbf{z}}_i, \mathbf{z}_k$ and $\dot{\mathbf{z}}_k$, where \mathbf{z}_i and $\dot{\mathbf{z}}_i$ represent the absolute displacement and the velocity of the node i and \mathbf{z}_k and $\dot{\mathbf{z}}_k$ are the absolute displacements and velocities of the N nodes that are connected to the node i , all of them in x, y and z axes. Furthermore, \mathbf{F}_{xi} , \mathbf{F}_{yi} and \mathbf{F}_{zi} are the external forces that are acting in the internal node i .

Expanding f_{xk} , f_{yk} and f_{zk} functions as Taylor series [11] and selecting only the first term, the dynamic equations for an internal node i are stated as:

$$\begin{aligned} m_i \ddot{z}_{xi} &= \sum_{k=1}^N (k_{xxki}(z_{xi} - z_{xk}) + k_{xyki}(z_{yi} - z_{yk}) + k_{xzki}(z_{zi} - z_{zk}) + c_{xxki}(\dot{z}_{xi} - \dot{z}_{xk}) + c_{xyki}(\dot{z}_{yi} - \dot{z}_{yk}) + c_{xzki}(\dot{z}_{zi} - \dot{z}_{zk})) + F_{xi} \\ m_i \ddot{z}_{yi} &= \sum_{k=1}^N (k_{yxki}(z_{xi} - z_{xk}) + k_{yyki}(z_{yi} - z_{yk}) + k_{yzki}(z_{zi} - z_{zk}) + c_{yxki}(\dot{z}_{xi} - \dot{z}_{xk}) + c_{yyki}(\dot{z}_{yi} - \dot{z}_{yk}) + c_{yzki}(\dot{z}_{zi} - \dot{z}_{zk})) + F_{yi} \\ m_i \ddot{z}_{zi} &= \sum_{k=1}^N (k_{zxki}(z_{xi} - z_{xk}) + k_{zyki}(z_{yi} - z_{yk}) + k_{zzki}(z_{zi} - z_{zk}) + c_{zxki}(\dot{z}_{xi} - \dot{z}_{xk}) + c_{zyki}(\dot{z}_{yi} - \dot{z}_{yk}) + c_{zzki}(\dot{z}_{zi} - \dot{z}_{zk})) + F_{zi} \end{aligned} \quad (2)$$

where z_{xi} , z_{yi} , z_{zi} and \dot{z}_{xi} , \dot{z}_{yi} , \dot{z}_{zi} are the absolute displacements and velocities of the node i and z_{xk} , z_{yk} , z_{zk} and \dot{z}_{xk} , \dot{z}_{yk} , \dot{z}_{zk} are the absolute displacements and velocities of the N nodes that are connected to the node i , all of them in x, y and z axes. On the other hand, k and c are coefficients related to the stiffness and damping values of the N bars that are in contact with node i .

The finite central difference method [10] is used to obtain the approximation of the displacement's first and second derivatives. Repeating the explained process (equation 2) for the other internal nodes and representing the expressions in matrix form, the substructural dynamic equation is stated as:

$$\begin{aligned}
 & \begin{bmatrix} z_{i1x}(n) \\ z_{i1y}(n) \\ z_{i1z}(n) \\ \dots \\ z_{inx}(n) \\ z_{iny}(n) \\ z_{inz}(n) \end{bmatrix} = -A_1 \begin{bmatrix} z_{i1x}(n-1) \\ z_{i1y}(n-1) \\ z_{i1z}(n-1) \\ \dots \\ z_{inx}(n-1) \\ z_{iny}(n-1) \\ z_{inz}(n-1) \end{bmatrix} - A_2 \begin{bmatrix} z_{i1x}(n-2) \\ z_{i1y}(n-2) \\ z_{i1z}(n-2) \\ \dots \\ z_{inx}(n-2) \\ z_{iny}(n-2) \\ z_{inz}(n-2) \end{bmatrix} \\
 & + B_{10} \begin{bmatrix} z_{ij1x}(n) \\ z_{ij1y}(n) \\ z_{ij1z}(n) \\ \dots \\ z_{ijnx}(n) \\ z_{ijny}(n) \\ z_{ijnz}(n) \end{bmatrix} + B_{11} \begin{bmatrix} z_{ij1x}(n-1) \\ z_{ij1y}(n-1) \\ z_{ij1z}(n-1) \\ \dots \\ z_{ijnx}(n-1) \\ z_{ijny}(n-1) \\ z_{ijnz}(n-1) \end{bmatrix} + B_{12} \begin{bmatrix} z_{ij1x}(n-2) \\ z_{ij1y}(n-2) \\ z_{ij1z}(n-2) \\ \dots \\ z_{ijnx}(n-2) \\ z_{ijny}(n-2) \\ z_{ijnz}(n-2) \end{bmatrix} + B_{21} \begin{bmatrix} F_{i1x}(n-1) \\ F_{i1y}(n-1) \\ F_{i1z}(n-1) \\ \dots \\ F_{inx}(n-1) \\ F_{iny}(n-1) \\ F_{inz}(n-1) \end{bmatrix}
 \end{aligned} \tag{3}$$

where Z_{i1x}, \dots, Z_{inz} and $Z_{ij1x}, \dots, Z_{ijnz}$ are the absolute displacements of all internal and interface nodes in x, y and z axes.

Equation (3) could be regarded as a VARX model [12], where Z_{i1x}, \dots, Z_{inz} corresponds to the endogenous variables and $Z_{ij1x}, \dots, Z_{ijnz}$ and F_{i1x}, \dots, F_{inz} correspond to the exogenous variables. A_k is a $n \times n$ endogenous coefficient matrix, where n denotes the number of internal DOFs of the substructure. The elements of A_k matrix are related to the physical properties of the bars that connect the internal nodes between them. On the other hand, B_k is a $n \times m$ exogenous coefficient matrix, where m denotes the number of interface DOFs of the substructure. The elements of B_k are related to the physical properties of the bars that connect one internal node to another interface node.

In the proposed method, the substructures are modelled as a VARX models. The state of each substructure is evaluated analyzing deviations in its estimated coefficient matrices (A_k and B_k) respect to its estimated coefficient matrices (A_k and B_k) in the healthy condition. Firstly substructural damages are detected and secondly the damages are located within the substructure.

3. Numerical results

A linear and time invariant two-dimensional lattice structure is studied in this section. The structure consists of stainless steel bars that are connected together by rigid joints and we assume that the forces could only be transmitted along the axial direction of the bars and the load could only be applied at the two ends of each bar. The structural behaviour is described by a lumped parameter model, where we assume that all object are rigid bodies and all interactions between the rigid bodies take place via springs.

In the studied case, the structure is modelled as a sixteen DOF mass-spring model (see figure 1). Furthermore, a ten DOF substructure is isolated from the general structure, where absolute displacement $Z_{6x}, Z_{6y}, Z_{7x}, Z_{7y}, Z_{8x}$ and Z_{8y} correspond to internal DOFs and absolute displacement $Z_{4x},$

z_{4y} , z_{5x} and z_{5y} correspond to interface DOFs. As shown in figure 1, none external force is applied within the substructure.

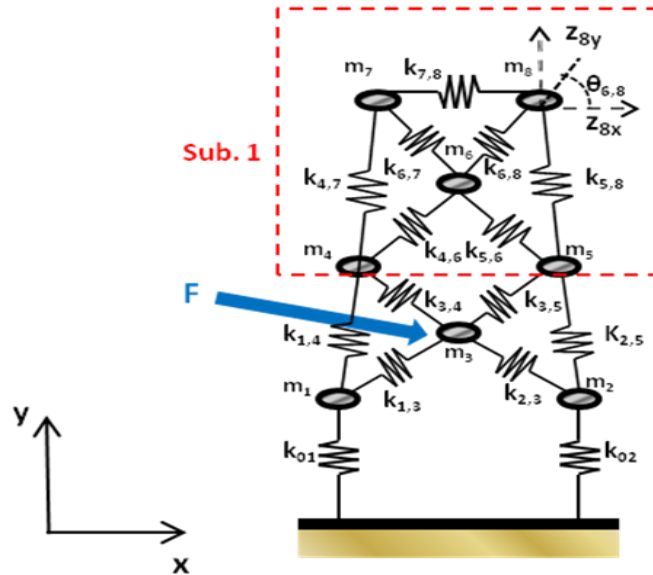


Figure 1. Isolated substructure in the structural model

Below, the dynamic equations for the internal node i are formulated:

$$m_i \ddot{z}_{xi} = \sum_{k=1}^N (k_{k,i} \cos^2 \theta_{k,i} (z_{xi} - z_{xk}) + k_{k,i} \cos \theta_{k,i} \sin \theta_{k,i} (z_{yi} - z_{yk})) \tag{4}$$

$$m_i \ddot{z}_{yi} = \sum_{k=1}^N (k_{k,i} \cos \theta_{k,i} \sin \theta_{k,i} (z_{xi} - z_{xk}) + k_{k,i} \sin^2 \theta_{k,i} (z_{yi} - z_{yk}))$$

where m_i represent the lumped mass of the internal node i and \ddot{z}_{xi} and \ddot{z}_{yi} are the absolute accelerations of the internal node i in x and y axes. The internal node i is supporting N internal forces, one force for each node connected to him. These forces depend on the stiffness and the angle respect to the x axis of the springs located between the internal node i and the N nodes that are connected to him, as well as the displacements of these nodes in x and y axes (z_{xi} , z_{yi} , z_{xk} , z_{yk}).

Following the procedure described in section 2, we get the VARX model. Equation (5) could be regarded as a four exogenous and six endogenous variables VARX model [12]. The exogenous variables are the measured absolute displacements in z_{4x} , z_{4y} , z_{5x} and z_{5y} and the endogenous variables are the measured absolute displacement in z_{6x} , z_{6y} , z_{7x} , z_{7y} , z_{8x} and z_{8y} .

$$\begin{bmatrix} z_{6x}(n) \\ z_{7x}(n) \\ z_{8x}(n) \\ z_{6y}(n) \\ z_{7y}(n) \\ z_{8y}(n) \end{bmatrix} = -A_1 \begin{bmatrix} z_{6x}(n-1) \\ z_{7x}(n-1) \\ z_{8x}(n-1) \\ z_{6y}(n-1) \\ z_{7y}(n-1) \\ z_{8y}(n-1) \end{bmatrix} - A_2 \begin{bmatrix} z_{6x}(n-2) \\ z_{7x}(n-2) \\ z_{8x}(n-2) \\ z_{6y}(n-2) \\ z_{7y}(n-2) \\ z_{8y}(n-2) \end{bmatrix} + B_1 \begin{bmatrix} z_{4x}(n-1) \\ z_{5x}(n-1) \\ z_{4y}(n-1) \\ z_{5y}(n-1) \end{bmatrix} \tag{5}$$

A_1 and A_2 are 6 x 6 endogenous coefficient matrices and B_1 is a 6 x 4 exogenous coefficient matrix. These matrices are related to the physical properties of the substructural bars (mass, stiffness and angle) and also depend on the used sampling period. In addition to this, equation 6 shows the dependence between each matrix element and the substructural stiffness values.

$$A_1 = \begin{bmatrix} f_{11}(k_{4,6}, k_{5,6}, k_{6,7}, k_{6,8}) & f_{12}(k_{6,7}) & f_{13}(k_{6,8}) & f_{14}(k_{4,6}, k_{5,6}, k_{6,7}, k_{6,8}) & f_{15}(k_{6,7}) & f_{16}(k_{6,8}) \\ f_{21}(k_{6,7}) & f_{22}(k_{4,7}, k_{6,7}, k_{7,8}) & f_{23}(k_{7,8}) & f_{24}(k_{6,7}) & f_{25}(k_{4,7}, k_{6,7}, k_{7,8}) & f_{26}(k_{7,8}) \\ f_{31}(k_{6,8}) & f_{32}(k_{7,8}) & f_{33}(k_{5,8}, k_{6,8}, k_{7,8}) & f_{34}(k_{6,8}) & f_{35}(k_{7,8}) & f_{36}(k_{5,8}, k_{6,8}, k_{7,8}) \\ f_{41}(k_{4,6}, k_{5,6}, k_{6,7}, k_{6,8}) & f_{42}(k_{6,7}) & f_{43}(k_{6,8}) & f_{44}(k_{4,6}, k_{5,6}, k_{6,7}, k_{6,8}) & f_{45}(k_{6,7}) & f_{46}(k_{6,8}) \\ f_{51}(k_{6,7}) & f_{52}(k_{4,7}, k_{6,7}, k_{7,8}) & f_{53}(k_{7,8}) & f_{54}(k_{6,7}) & f_{55}(k_{4,7}, k_{6,7}, k_{7,8}) & f_{56}(k_{7,8}) \\ f_{61}(k_{6,8}) & f_{62}(k_{7,8}) & f_{63}(k_{5,8}, k_{6,8}, k_{7,8}) & f_{64}(k_{6,8}) & f_{65}(k_{7,8}) & f_{66}(k_{5,8}, k_{6,8}, k_{7,8}) \end{bmatrix} \quad (6)$$

$$A_2 = I$$

$$B_1 = \begin{bmatrix} g_{11}(k_{4,6}) & g_{12}(k_{5,6}) & g_{13}(k_{4,6}) & g_{14}(k_{5,6}) \\ g_{21}(k_{4,7}) & 0 & g_{23}(k_{4,7}) & 0 \\ 0 & g_{32}(k_{5,8}) & 0 & g_{34}(k_{5,8}) \\ g_{41}(k_{4,6}) & g_{42}(k_{5,6}) & g_{43}(k_{4,6}) & g_{44}(k_{5,6}) \\ g_{51}(k_{4,7}) & 0 & g_{53}(k_{4,7}) & 0 \\ 0 & g_{62}(k_{5,8}) & 0 & g_{64}(k_{5,8}) \end{bmatrix}$$

In this work, the structure is excited in the third mass (outside the substructure) by a Gaussian white noise and the displacements are recorded for each substructural DOF using a data sampling frequency of 1000 Hz. Later, the substructural VARX model is estimated by the Multivariable Least-Square estimator (MLS) method [12] for a healthy state and for the damaged scenarios. All considered damages are stiffness losses of a specific spring within the structure. Three different damage severities (5%, 10% and 20%) and six different damage locations are evaluated. In some of them, the damaged springs are within the substructure ($k_{4,6}$, $k_{4,7}$, $k_{6,7}$, $k_{6,8}$) and in the others, they correspond to external spring ($k_{1,3}$, $k_{2,5}$).

As we could see in equation (6), the elements of matrices A_1 and B_1 are function, among other things, of the stiffness of the substructural springs. Matrix A_1 depends on the state of the springs that are located between the internal nodes ($k_{6,7}$, $k_{6,8}$, $k_{7,8}$) and matrix B_1 depends on those springs located between the internal and interface nodes ($k_{4,6}$, $k_{4,7}$, $k_{5,6}$, $k_{5,8}$).

In this work we evaluate the state of the whole substructure. For this purpose, firstly the substructural VARX model of the healthy state is estimated. Later, the substructural VARX model is reestimated in each new scenario and the state of the substructure is evaluated in these new scenarios comparing these updated A_1 and B_1 matrices and the healthy ones. In the case of substructural damages, the damages will be firstly detected and later located in one of the substructural spring depending on the varied elements within A_1 and B_1 matrices. On the other hand, if external springs are damaged, the elements of matrices A_1 and B_1 will not change, so these external damages will not be detected and located within the substructure.

Table 1 shows which elements of A_I and B_I matrices depend exclusively on the properties of the substructural springs. In this work, only the variations of these elements (see table 1) are analyzed and the state of each substructural spring is evaluated depending on this analysis. For example, the stiffness of the spring that joins nodes 5 and 6 ($k_{5,6}$) affects exclusively in four elements within the matrix B_I ($B_{I(1,2)}$, $B_{I(1,4)}$, $B_{I(4,2)}$ and $B_{I(4,4)}$). For this reason, the variations of these four elements are analyzed to determine if the spring $k_{5,6}$ is damaged or not. In the present study, the damage severity for each spring is given by the mean variation of the elements that should be analyzed (see table 1).

Table 1. Analyzed elements within A_I and B_I matrices

Internal spring	A_I elements	B_I elements
$k_{4,6}$	-	(1,1),(1,3),(4,1),(4,3)
$k_{4,7}$	-	(2,1),(2,3),(5,1),(5,3)
$k_{5,6}$	-	(1,2),(1,4),(4,2),(4,4)
$k_{5,8}$	-	(3,2),(3,4),(6,2),(6,4)
$k_{6,7}$	(1,2),(1,5),(2,1),(2,4) (4,2),(4,5),(5,1),(5,4)	-
$k_{6,8}$	(1,3),(1,6),(3,1),(3,4) (4,3),(4,6),(6,1),(6,4)	-
$k_{7,8}$	(2,3),(2,6),(3,2),(3,5) (5,3),(5,6),(6,2),(6,5)	-

Regarding to the results, for external damages (reducing $k_{1,3}$ and $k_{2,5}$ values), the estimated stiffness modification for all substructural springs are almost zero, so the method determines that the substructure is healthy. For internal damages (reducing $k_{4,6}$, $k_{4,7}$, $k_{6,7}$ and $k_{6,8}$ values), the estimated stiffness modifications are shown in figure 2. In these four scenarios, the substructural damages are firstly detected and later located in the proper spring. Furthermore, the results show that the method estimates the severity of the damage.

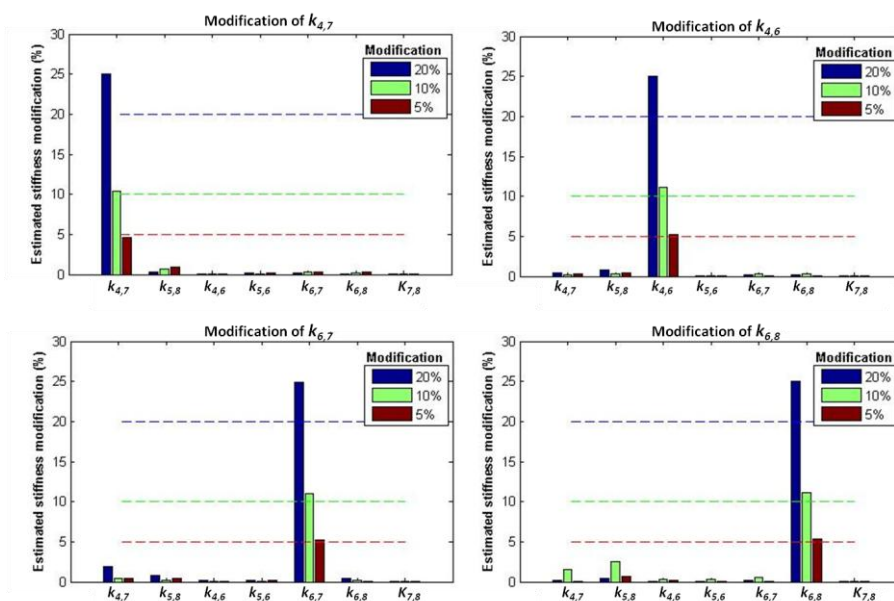


Figure 2. Estimated stiffness modification for each substructural spring (internal damages)

4. Conclusions

This paper proposes a novel SHM method to locate damages in multidimensional structures. A substructure of interest is isolated by a substructuring method and a VARX model of the isolated substructure is obtained. The analysis of the estimated VARX model is carried out in order to evaluate the health of the isolated substructure. It is not necessary to have any theoretical model and only the measured displacement data is required to estimate the isolated substructure's VARX model.

A linear and time invariant model of a two-dimensional lattice structure is simulated to evaluate the proposed method. The results show that the method not only allows detecting damages within the substructure, because it also allows estimating their location and their severity.

The proposed method is also suited for three dimensional lattice structures, where the number of element's connections increases. Our research group is already applying this method in a laboratory lattice structure and the results will be published soon.

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