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An Optimal Power-Splitting Strategy for Hybrid Storage Systems

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Abstract—The wide deployment of inverter-based resources is changing the grid characteristics and demanding rapid changes in the control and operation of power grids. Storage is becoming a key component of this transition, as it can provide a wide palette of services, from fast frequency response to congestion management. This implies a broad range of requirements for the storage units, in terms of energy, power, response times, etc. Hybrid storage systems, combining units with different properties, such as supercapacitors, high-energy or high-power batteries, represent a natural choice that can cover many grid services. These services do not only require active power but also reactive power, and ideally they will be provided by the storage system while optimizing its overall efficiency. In this work we formally derive active and reactive power splitting strategies that minimize the operation losses among all inverters of the hybrid storage system. These strategies are computationally light and do not rely on complex optimizers, thereby facilitating their implementation in embedded systems. A realistic example depicts the clear improvement of this approach against simple power splitting strategies. While our focus lies on hybrid storage systems, the formulation can be applied as well to hybrid power plants, since their power losses can be characterized by similar mathematical functions.

Index Terms—Hybrid Energy Storage Systems, Efficient Power Electronics Converters, Grid Services, Energy Management Systems

I. INTRODUCTION

The ongoing energy transition requires new paradigms and new assets capable of addressing novel grid needs. Storage units can address several of these system requirements, even more when these units are composed of different sources, such as batteries, supercapacitors, electrolyzers, thermal storage or pumped hydro plants [?]. In this spirit, hybrid power plants can leverage heterogeneous capabilities in a collocated manner, in order to identify and exploit synergies between different sources, such as PV and wind. Among all these new system needs, the transition from conventional generation to inverter-based generation is motivating the provision of ancillary services from units that interface the grid via power electronics. This implies that inverters will be required to provide a certain amount of active and reactive power, based on for instance frequency and voltage [?], [?]. Ideally these services are provided in an optimal way, that is, maximizing the efficiency of the units composed of inverters and storage. At the same time, since storage units are becoming larger, they

are now typically composed of different units with different characteristics to address different needs [?], e.g., they can be composed of a high energy and a high power module and their correspondent submodules. An example of such a setup is shown in Fig ??.

Improving efficiency in power converters has been for many years an important focus of interest for academia and industry, as nowadays most new energy sources are connected to the grid via power converters. A large amount of work has focused on improving the overall efficiency of the inverter. However, the efficiency at low loads is still significantly lower than at nominal power, mainly due to switching losses that are not proportional to the output power. In addition, improvement of the efficiency at partial load with classical converter topologies is considered very challenging [?]. A somewhat flat efficiency curve could be achieved by means of modular topologies, where modules can be deactivated whenever possible. In this sense, the work in [?] suggests a strategy to decide when to de/activate modules and how to split the setpoints among the modules. This paper extends these ideas in two directions, first by considering heterogeneous inverters with different efficiency curves, and second including the dispatch of reactive power. Moreover, the derived power splitting conditions are proven to be optimal, in the sense that the overall losses are minimized. The main focus is on minimizing inverter losses, but the corresponding battery losses can be included as well using this formulation, since they are quadratic in nature, as in the case of inverters [?].

It is important to emphasize that the efficient operation of inverters providing ancillary services is very relevant, as these units operate most of the time providing a relatively small amount of power, given that extreme grid events only occur rarely. Indeed, voltage or frequency support would only require high power under very large frequency variations or in very weak grids under high loading.

This paper is organised as follows. First, the optimisation problem is posed, the assumptions are defined and the type of solutions to be derived are motivated. Next, for didactic purposes, the simple case of active power dispatch between homogeneous inverters is introduced, and then the problem is extended to reactive power and heterogeneous inverters. Finally, a discussion on how to generalize these results to hybrid power plants is presented.

Hybrid energy storage system composed of n different modules.

II. OPTIMAL POWER SPLITTING STRATEGIES

We consider here the problem of providing a certain desired active and reactive power with a hybrid storage system, com-

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posed of modules of different characteristics interfacing the grid via power converters. The goal is to split a reference for P and Q among the modules while maximizing the efficiency of the converters, that is, minimizing the losses. Mathematically speaking, the cost function to minimize is given by:

$$\begin{aligned} & \min_{P_i, Q_i} \sum_i S_{\text{Loss}_i} \\ \text{s.t. } & P_i^2 + Q_i^2 \leq S_{\text{max}_i}^2 \quad \forall i = 1, \dots, n, \\ & \sum_i P_i = P_{\text{ref}}, \sum_i Q_i = Q_{\text{ref}} \end{aligned} \quad (1)$$

where P_i, Q_i are the active and reactive power to be provided by each module of each storage system, S_{Loss_i} are the losses of each module, S_{max_i} is the maximum apparent power of each module, and $P_{\text{ref}}, Q_{\text{ref}}$ are the dispatch values or setpoints for the plant. Notice that a priori there is no constraint in the sign of P_i or Q_i , as the module could be providing or absorbing active or reactive power. In most cases, the losses of a module depend on the current; assuming that the terminal voltage of the inverter is constant, the losses can be written as a function of the apparent power S [?]. For simplicity, at this point we focus on the converter efficiency; nonetheless, the energy sources behind can also be included within this framework, as shown afterwards. The converter losses typically exhibit a quadratic behaviour whenever the module is in operation, as reported in the literature [?], [?], [?]:

$$S_{\text{Loss}_i}(S_i) \approx b_i \cdot (c_{0_i} + c_{1_i} S_i + c_{2_i} S_i^2), \quad b_i \in \{0, 1\} \quad (2)$$

with $c_0, c_1, c_2 \geq 0$ and where the boolean variable b_i denotes if the i -th module is active or not. Publicly available datasheets confirm the validity of this assumption [?]. At the same time, losses from batteries or conventional sources are traditionally characterised by quadratic functions. Therefore, both the converter and the energy sources losses can be integrated, where now the coefficients represent the combined affine, linear and quadratic losses of the inverter and the battery (see e.g. [?]).

For this type of applications, computationally simple solutions are sought, ideally explicit solutions, since the power splitting algorithm is expected to be implemented in an embedded device. Therefore, iterative algorithms are to be avoided. The use of look-up tables would be acceptable, provided that their size is limited. For didactic purposes, we will divide the original problem in subproblems in order to better understand the optimization setup and the constraints.

A. The case of P -dispatch among homogeneous modules

We first consider only the case of active power dispatch, assuming all power modules are homogeneous and therefore exhibit the same losses curve, i.e., $c_{0_i} = c_0$ for all $i = 1, \dots, n$. Then, problem (??) becomes:

$$\begin{aligned} & \min_{P_i} \sum_i b_i \cdot (c_0 + c_1 |P_i| + c_2 P_i^2) \\ \text{s.t. } & P_i^2 \leq S_{\text{max}_i}^2 \quad \forall i, \sum_i P_i = P_{\text{ref}}, \quad b_i \in \{0, 1\} \end{aligned} \quad (3)$$

This formulation can be seen as a constrained mixed-integer problem (an NP-hard problem), which can be very cumbersome and may require of some heuristic approach to find a suboptimal solution in a reasonable amount of time. Instead, we will divide the problem in several steps. First, the problem

is relaxed ignoring the binary variables, then an optimal strategy is devised for the binary decision variables. The absolute value can also be removed since the sign of all P_i needs to be the same as the sign of P_{ref} : for all solutions of interest we do not have some modules providing power and other ones absorbing it, as it will lead to more losses in all cases. Thus, for convenience and without loss of generality, we will assume all powers to be positive, that is, $P_i \geq 0$, for all i . The formulation in (??) is then reframed as:

$$\min_{P_i} \sum_i c_1 P_i + c_2 P_i^2 \quad (4)$$

$$\text{s.t. } \sum_i P_i = P_{\text{ref}} \quad (5)$$

$$P_i \geq 0, \quad \forall i \quad (6)$$

$$P_i^2 \leq S_{\text{max}_i}^2, \quad \forall i \quad (7)$$

If we ignore for the time being the inequality constraints in (??) and (??), this optimization problem in (??)-(??), typically known as equality constrained convex quadratic minimization, has a simple explicit solution (and unique) that can be computed using Lagrange multipliers, given by:

$$P_i = P_{\text{ref}}/n \quad (8)$$

with n denoting the number of active modules. That is, all modules should equally split the power setpoint. This solution actually coincides with the well-known strategy in [?]. Clearly this solution of the relaxed problem also satisfies the constrained problem, as any admissible P_{ref} is always expected to be less than the sum of the ratings of each converter, and P_i has the same sign as P_{ref} . Hence the inequalities (??) and (??) are implicitly satisfied for realistic values of P_{ref} .

Now we focus on the binary variables, to decide how many modules should be active, knowing that the optimal solutions is to split power equally among the active modules. Given the affine term in the losses, it is clear that for low power values, only one module should be active. Since the modules are identical, the switching should occur when the losses of using one module become larger than using two modules:

$$P_{\text{Loss}_1}(P_{\text{ref}}) = P_{\text{Loss}_1}(P_{\text{ref}}/2) + P_{\text{Loss}_2}(P_{\text{ref}}/2)$$

This can be easily generalized to consider multiple modules, and straightforward computations dictate that the activation of the i -th module should occur whenever the following equality holds:

$$P_{\text{ref}} = \sqrt{\frac{c_0}{c_2} i(i-1)}, \quad i > 0$$

Switching needs to occur as well when the aggregated rating of all active modules is less than the dispatch value:

$$P_{\text{ref}} = \min(i S_{\text{max}}, \sqrt{\frac{c_0}{c_2} i(i-1)}), \quad i > 0 \quad (9)$$

Hence, the optimal strategy for the case of homogeneous modules and considering just active power is given by equations (??) and (??).

B. The case of P-dispatch among heterogeneous modules

In this case, the coefficients of each loss curve are different:

$$\min_{P_i} \sum_i b_i \cdot (c_{0_i} + c_{1_i}|P_i| + c_{2_i}P_i^2) \quad (10)$$

$$s.t. P_i^2 \leq S_{\max_i}^2 \quad \forall i, \quad \sum_i P_i = P_{\text{ref}}, \quad b_i \in \{0, 1\}$$

Nonetheless, we can follow similar steps relaxing the original problem as in the previous subsection (by excluding inequalities (??) and (??)), and reformulating the problem as a quadratic cost function under linear constraints. As before, lengthy but straightforward computations using Lagrange multipliers take us to the following unique solution:

$$P_i = \frac{1}{2c_{2_i}}(-c_{1_i} + \frac{1}{\sum_i \frac{1}{2c_{2_i}}}(P_{\text{ref}} + \sum_i \frac{c_{1_i}}{2c_{2_i}})) \quad (11)$$

This equation shows that the optimal power setpoint is an affine function of the reference power, where the linear term is the same for all modules. Thus the difference between the power at different modules is just an offset. Moreover, these coefficients in (??) can be precomputed beforehand. Since the relaxed problem does not include the inequality constraint on the power as in (??), the solutions (??) could lead to negative values, which corresponds to at least one module charging and one discharging. This implies that the omitted constraint $P_i \geq 0$ is active and this module should not be used. Likewise, it could lead to values of P_i above the rated power. This is however unlikely to happen, given that for most modular systems it is optimal to increase the number of active modules before the maximum rated power of one module is reached (see for instance [?]). In any case, should this happen, it can be integrated using complementary slack variables and solving the Karush-Kuhn-Tucker conditions [?], by fixing the value of the corresponding module either to 0 or $S_{\max}$ and applying again (??) with the remaining modules, until all values are within limits.

Unlike in the previous section, switching and module activation becomes convoluted: the expressions can be computed, but they are cumbersome, and all combinations need to be verified. Two possibilities can be contemplated here, taking advantage of the already derived expressions of the optimal setpoints for each module: a) the losses of each combination of modules can be computed, and afterwards the configuration with the minimum losses is selected; b) only the points at which activation needs to occur are analysed. The latter seems to be less computationally intensive and will be further explored here. Since the losses are assumed to be quadratic, there will be only a finite amount of points where activation or deactivation will be required, and can be precomputed offline.

This activation/deactivation of modules occur when the losses under a given configuration are equal to the losses under another configuration. The optimal losses given a number of active modules can be computed using the expression in (??). This will lead us to a quadratic expression on P_{ref} , regardless of the number of modules. As a simple example, we consider the case of two modules with different loss curves, taken from publicly available data from the manufacturer SMA [?] and shown in Fig ?? . We can have three different configurations: either module A or B is active, or both modules are active.

This corresponds to two configuration switching points. The analytical expression for the activation of module B and deactivation of module A can be easily computed from the following second order equation:

$$(c_{2_A} - c_{2_B})P_{\text{ref}}^2 + (c_{1_A} - c_{1_B})P_{\text{ref}} + c_{0_A} - c_{0_B} = 0$$

In the case of the activation of both modules, the expression becomes more convoluted but it is still quadratic, and therefore, straightforward to derive and to solve.

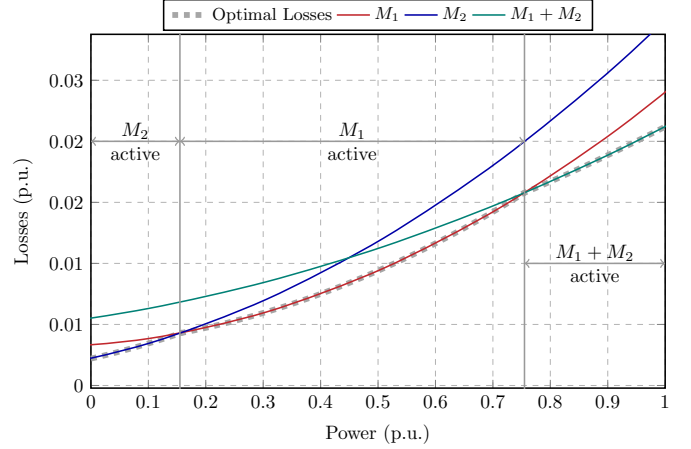


Fig. 1. Power losses for the 3 different configurations: only M_1 active, only M_2 active, both M_1 and M_2 active.

This line of thought can be generalized: given the optimal dispatch power for a fixed number of modules as in (??), we can rewrite the optimal costs of one configuration as:

$$\sum_{i \in \Omega} c_{0_i} + c_{1_i}|P_{\text{ref}}^*| + c_{2_i}P_{\text{ref}}^{*2} = \sum_{i \in \Omega} \alpha_{0_i} + \alpha_{1_i}|P_{\text{ref}}| + \alpha_{2_i}P_{\text{ref}}^2$$

where α_{ij} can be easily derived from (??), and the set Ω is a subset of the all possible combinations of the modules. Therefore, all the intersection points can be precomputed as:

$$\sum_{i \in \Omega} (\alpha_{0_i} + \alpha_{1_i}|P_{\text{ref}}| + \alpha_{2_i}P_{\text{ref}}^2) = \sum_{j \in \Delta} (\alpha_{0_j} + \alpha_{1_j}|P_{\text{ref}}| + \alpha_{2_j}P_{\text{ref}}^2)$$

where $\Omega, \Delta \in 2^X$, and 2^X is the superset that includes all combinations of the n modules. While the problem is combinatorial in nature, it is important to emphasize that these type of hybrid storage units will not be composed of many heterogeneous sources, and therefore computing all these switching points is not computationally demanding.

Finally, we would like to point out that this optimization problem in (??) is identical to the classical lossless economic dispatch problem in power systems. In our application, the number of variables tend to be smaller, but at the same time the available computational power is much lower.

C. The case of P, Q -dispatch among homogeneous modules

We consider now as well the case of reactive power dispatch. The optimization problem can be written as:

$$\begin{aligned} \min_{P_i, Q_i} \sum_i b_i \cdot (c_0 + c_1 S_i + c_2 S_i^2) \\ \text{s.t.} \quad \sum_i P_i = P_{\text{ref}} \quad \sum_i Q_i = Q_{\text{ref}} \\ S_i^2 \leq S_{\text{max}}^2 \quad \forall i, \quad b_i \in \{0, 1\} \end{aligned}$$

The cost function is no longer quadratic in the decision variables, since $S_i = \sqrt{P_i^2 + Q_i^2}$, and no straightforward solution can be found. Nonetheless, to derive an analytical solution we rewrite the system in the so-called polar coordinates, that is, $P_i = S_i \cos \phi_i$ and $Q_i = S_i \sin \phi_i$. In this way the cost function becomes again quadratic, and the nonlinearities are moved into the constraints:

$$\min_{S_i, \phi_i} \sum_i b_i \cdot (c_0 + c_1 S_i + c_2 S_i^2) \quad (12)$$

$$\text{s.t.} \quad S_i^2 \leq S_{\text{max}}^2 \quad \forall i, \quad b_i \in \{0, 1\} \quad (13)$$

$$\sum_i S_i \cos \phi_i = P_{\text{ref}}, \quad \sum_i S_i \sin \phi_i = Q_{\text{ref}} \quad (14)$$

As before, we tackle the problem in two steps, first ignoring the binary variables and removing the constraint on maximum ratings (which will be checked afterwards). We can construct the Lagrangian as:

$$\begin{aligned} L = \sum_i (c_0 + c_1 |S_i| + c_2 S_i^2) + \lambda_1 (S_i \cos \phi_i - P_{\text{ref}}) \\ + \lambda_2 (S_i \sin \phi_i - Q_{\text{ref}}) \end{aligned}$$

The derivative of the Lagrangian with respect to the angles ϕ_i gives us the following set of equations:

$$\frac{\partial L}{\partial \phi_i} = -\lambda_1 S_i \sin \phi_i + \lambda_2 S_i \cos \phi_i$$

Since all extrema need to satisfy $\frac{\partial L}{\partial \phi_i} = 0$, it can be concluded that:

$$\frac{\lambda_2}{\lambda_1} = \frac{\sin \phi_i}{\cos \phi_i}, \quad \forall i \quad (15)$$

Since (15) needs to hold for all modules, it can be stated that $\frac{\sin \phi_i}{\cos \phi_i} = \frac{\sin \phi_j}{\cos \phi_j}$, for any two modules i and j . Given that the sign of active and reactive power should be the same for all modules, the angles need to be then identical for all modules. Therefore, the original problem can be simplified in different ways; for instance, we can substitute the constraints in (14) on P_i, Q_i for a constraint in S_i that no longer depends on $\cos \phi_i$, e.g.:

$$P_{\text{ref}} = \sum_i S_i \cos \phi_i = \cos \phi \sum_i S_i = \cos \phi S_{\text{ref}}$$

Proceeding in a similar way for the constraint on Q , the original problem can be rewritten as:

$$\begin{aligned} \min_{S_i} \sum_i b_i \cdot (c_0 + c_1 |S_i| + c_2 S_i^2) \quad (16) \\ \text{s.t.} \quad S_i^2 \leq S_{\text{max}}^2 \quad \forall i, \quad \sum_i S_i = S_{\text{ref}} \quad b_i \in \{0, 1\} \end{aligned}$$

where ϕ_i is no longer a decision variable, and $S_{\text{ref}} = \sqrt{P_{\text{ref}}^2 + Q_{\text{ref}}^2}$. Notice that this is the same problem as for the

case of active power in subsection ??, whose solution leads to equal power splitting among the modules as in (??):

$$\begin{aligned} S_i^* &= S_{\text{ref}}/n \\ P_i^* &= S_i^* \cos \phi^* = S_{\text{ref}}/n \cdot P_{\text{ref}}/S_{\text{ref}} = P_{\text{ref}}/n \\ Q_i^* &= S_i^* \sin \phi^* = S_{\text{ref}}/n \cdot Q_{\text{ref}}/S_{\text{ref}} = Q_{\text{ref}}/n \end{aligned} \quad (17)$$

Likewise, the activation of the modules is given by a similar expression as in (??):

$$S_{\text{ref}} = \min(i S_{\text{max}}, \sqrt{\frac{c_0}{c_2} i(i-1)}), \quad i > 0 \quad (18)$$

Remark. In some cases, depending on the point of operation in P and Q , the DC link voltage or the type of modulation, the inverters might experience an asymmetrical limit in terms of apparent power [?]. In that case, the constraint $S_i^2 \leq S_{\text{max}}^2$ is no longer useful and needs to be replaced by a tighter bound, e.g., taking the form of an ellipsoidal constraint as $P^2 + (Q + \alpha)^2 \leq \beta$, for some $\alpha, \beta \geq 0$. Computationally speaking, it may be more beneficial to characterize the admissible region by adding a set of linear constraints describing the restricted operation of the inverters. The problem can be tackled using the Karush-Kuhn-Tucker conditions, as in Section ?. The presence of harmonics and/or unbalance may also influence the maximum apparent power that the inverter can provide.

D. The case of P, Q -dispatch among heterogeneous modules

Finally, we consider the most general case: splitting setpoints of P and Q considering different characteristic curves for each module. As in the previous section, we can rewrite the formulation using polar coordinates, after simplifying the problem by removing boolean variables and the maximum rating constraints. Following the same reasoning and constructing the Lagrangian, we can see that (??) does not depend on the cost function and therefore on the coefficients of each curve. Hence, also in the case of heterogeneous power modules, the optimal choice is to have all converters with the same power factor. Applying a similar reasoning as before, we reformulate the problem as:

$$\begin{aligned} \min_{S_i} \sum_i (c_{0i} + c_{1i} |S_i| + c_{2i} S_i^2) \quad (19) \\ \text{s.t.} \quad S_i^2 \leq S_{\text{max}_i}^2 \quad \forall i, \quad \sum_i S_i = S_{\text{ref}} \end{aligned}$$

The solution is therefore identical as in subsection ??, replacing active power with apparent power:

$$S_i = \frac{1}{2c_{2i}} \left(-c_{1i} + \frac{1}{\sum_i 1/2c_{2i}} (S_{\text{ref}} + \sum_i \frac{c_{1i}}{2c_{2i}}) \right) \quad (20)$$

Once the apparent power setpoints are computed for each module, the corresponding values for P and Q for each module can be computed as well, as the optimal power factor is identical for all modules.

In the case of switching and how to decide which module is active, we can also derive beforehand quadratic equations as in subsection ?? replacing P_{ref} with S_{ref} , as the cost function is exclusively impacted by the apparent power.

TABLE I
LOSS COEFFICIENTS FOR THE INVERTERS OF THE TWO MODULES

	c_0	c_1	c_2
M_1	0.0033	0.0037	0.0170
M_2	0.0022	0.1070	0.0170

III. VALIDATION

IEEE 13 Bus System with a hybrid storage system connected at the end of the feeder.

To show the benefits of the proposed approach we consider the case of a hybrid storage system equipped with two heterogeneous modules, providing frequency and voltage support to a weak grid. As an example of a weak grid, the IEEE 13 node test feeder [?] is used, with the storage system at the end of one of the feeders, as depicted in Fig ??.

The two inverters are rated each at 100 kW and the coefficients of their loss characteristics can be found in Table ?. These values correspond to the curves previously presented in Fig ?? and were obtained from real inverters, as shown in [?].

To construct a realistic scenario, frequency measurements of the European grid sampled at 100ms were used for the input frequency profile. For the active power setpoints, the inverters have implemented primary regulation, i.e., the power is proportional to the frequency deviations. Likewise, for the reactive power setpoints the storage unit is providing voltage support in the form of a proportional controller (i.e., $Q_{\text{ref}} = k_V \cdot \Delta V$, with $k_V > 0$ being the voltage droop gain and ΔV the voltage deviation from a desired nominal value). The optimization problem is solved also at $t = 100\text{ms}$, although it could be solved at a slower rate, to avoid continuous activation and deactivation of certain modules. Moreover, a hysteresis band could be added if needed in order to avoid chattering around certain setpoint values.

Using the system described above, we evaluated the performance of the proposed method and the obtained energy savings. We compare the results of the proposed optimal strategy against a default approach that splits equally between the two modules the active and reactive setpoints, that is, $P_{\text{default}} = P_{\text{ref}}/2$ and $Q_{\text{default}} = Q_{\text{ref}}/2$. It is important to emphasize that the simple approach that we use as benchmark is a fairly good assumption, since, as shown in this paper, it happens to be an optimal policy should the modules characteristics be identical.

Fig. ?? shows one hour of the 7-day simulation, with the top plot representing the total power demand of the system. The data is presented scaled to the module per unit system, thus, the maximum power of the entire system is 2 p.u. The next two plots show the activation and the power provided by the two modules. Finally, the last plot shows in percentage the saved losses when compared to the default approach. For this particular hour, the proposed approach is able to reduce the losses by an average of 30.9%.

The reduction in losses is achieved by means of two factors. On the one hand, given the high values of c_0 for both modules, the strategy uses only one inverter whenever possible. As highlighted in both Fig ?? and Fig ??, there are two inflexion

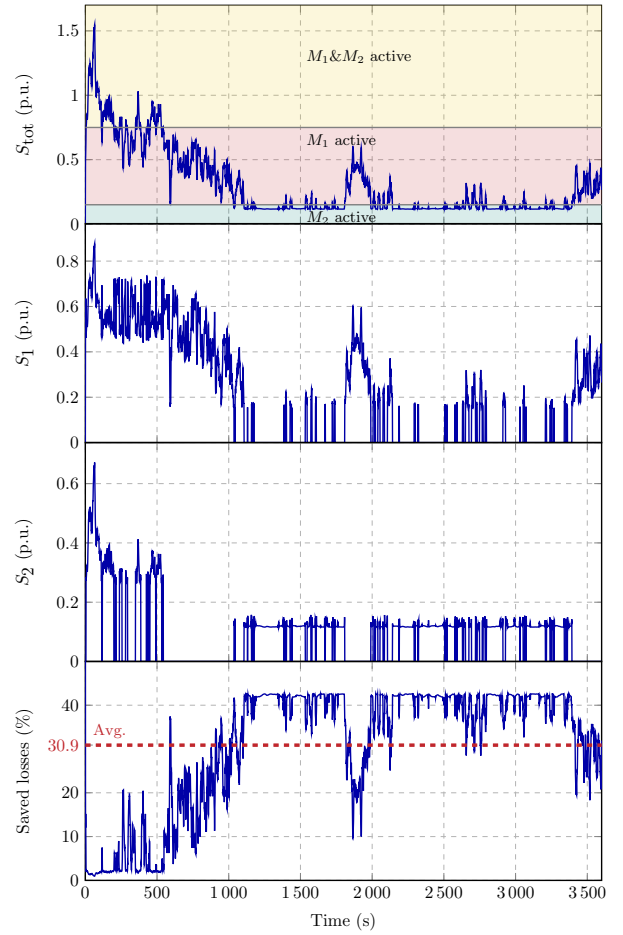


Fig. 2. One hour of simulation data under the optimal power sharing strategy. Plots from top to bottom: total power demand of the system, power S_1 provided by M_1 , power S_2 provided by M_2 , relative improvement with respect to the default power sharing strategy.

points for our system establishing three operation areas. In the first one, corresponding to power references smaller than 0.15 p.u., the strategy engages only M_2 . For power references between 0.15 p.u. and 0.75 p.u., only M_1 is enabled and for power references larger than 0.75 p.u., both M_1 and M_2 are enabled. These particular thresholds are only valid for the chosen system modules. For modules with smaller losses at low power, the strategy may activate both modules even for low and medium power values. On the other hand, when both modules are active more power is assigned to the module with smaller c_1/c_2 ratio, according to (?). In fact, the difference between the modules power references is just an offset that depends on the loss coefficients c , that can be precomputed. Hence, as mentioned before, the proposed strategy does not involve convoluted computations.

Finally, in Fig. ?? we show the cumulative losses over the entire time span which we considered for our analysis. It can be seen that at the end of the 7 days, 41.79 kWh could be saved by employing the proposed optimal power sharing strategy rather than the default one, which accounts for around 32% of the total losses during this period.

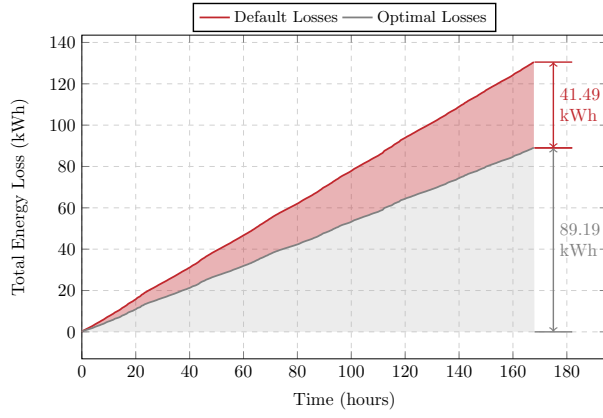


Fig. 3. Cumulative energy loss over seven days.

IV. DISCUSSION

A. Application to hybrid power plants

As mentioned before, similar strategies can be applied for hybrid power plants where the system needs to operate in an optimal manner by splitting setpoints among the available plants, as long as the losses can be represented by a quadratic function, which is typically the case (see for instance [?]). For instance, for conventional machines, quadratic costs are typically considered for the problem of economic dispatch. However, in some cases the problem setup vastly differs from the one presented here, in several directions: first, if one of the units is interfacing the grid via a synchronous machine, it will be mainly responsible for voltage control and reactive power support from power converters will not be needed. And even if it is needed, the capabilities of synchronous machines largely exceed those of the inverters. Second, given ramp rates and turn on/off costs of certain conventional sources, it is not desired to be changing setpoints very often, which implies that the splitting algorithm does not need to be executed at a high rate, and therefore complex optimization problems can be solved as computational complexity is less of an issue.

B. Future work

As mentioned throughout the paper, this framework can be extended to include the losses of the energy sources and to consider other types of hybrid plants. Given that the strategy attempts at minimizing the losses, it can be the case that the most efficient module is used very frequent, and therefore the lifetime and/or the state of charge (SoC) of that module gets penalized. In our example this did not happen in a significant manner since each system was more efficient in a different power range (see Fig ??). Nonetheless, whenever necessary this can be compensated by introducing a regularization term that depends on the actual SoC, or the amount of cycles experienced by a module. In the case of homogeneous systems, modules are interchangeable so adding extra considerations such as SoC constraints or number of cycles is fairly straightforward and can be seen as an implicit balancing method. Notice that, while many techniques that acknowledge SoC values resort to optimization over a time horizon (as in

model predictive control), in the case of ancillary services provision this is not practical, since estimating the evolution of frequency in the grid is in general not possible. Hence, a greedy optimization that does not consider a time window is still a very reasonable approach. Even with this regularization term, it is likely that balancing between the modules might be needed. This process can be integrated into the optimization problem by adding a linear constraint on the active power setpoints for each module.