Vibration attenuation of conductive beams by inducing eddy currents

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Abstract. The increasing requirements for structural vibration control in many industries, require innovative attenuation techniques. In this work, the phenomenon of eddy currents is proposed to reduce the vibration of conductive and non-magnetic beam-like structures without modifying the system, neither the weight nor the stiffness. The motion of a conductive material in a stationary magnetic field induces eddy currents, which in turn generate a repulsive force and attenuate the vibration. In this study, the vibrational response of a thin aluminium beam under a partial and stationary magnetic field is analysed. The influence of the eddy currents is experimentally studied in the bandwidth from 0 to 1 kHz and a preliminary numerical model is proposed. The results show the vibration of all the length of the beam can be attenuated by inducing eddy currents, whereas the natural frequencies of the system remain unmodified. The attenuation of the vibration is more remarkable at low frequencies and when the position of the magnetic field coincides with a maximum vibration of a mode.

1. Introduction

The structural vibration control is a fundamental concern in industries such as aeronautical, automotive, rail, marine and construction, due to the continuously increasing requirements. There are different techniques aimed at the control of structural vibrations, classified into active, semiactive, adaptive and passive techniques [1]. When adopting many of these techniques the characteristics of the system, specifically the weight and stiffness, are modified.

When a conductive material is exposed to a time varying magnetic field, eddy currents are induced in the conductive material. The time varying magnetic field can be obtained by a relative movement between the conductive material and magnetic field source, which can be the vibration of a beam-like structure under a steady magnetic field. The induced eddy currents circulate in such a way that they generate a magnetic field with opposite polarity to the change in flux and they cause a repulsive force that attenuates the vibration.

The concept of eddy currents has been known for long time and many authors have proposed different uses of eddy currents in braking and damping systems [2]. Matsutzaki et al [3,4] proposed suppressing the vibration of a beam, periodically magnetized along its length, using electromagnetic forces generated by a current passing between the magnetized sections. Cheng et al [5] damped the vibration of a cantilever beam by means of a shunted electromagnetic transducer. Sodano et al [6] analysed the vibration suppression capabilities of a cantilever beam by placing a magnet perpendicular to the beam motion and attaching a conductive sheet in the beam tip. In later works, Sodano et al [7,8] proposed the use of two magnets, facing each other with same polarity, or the active movement of the magnet to increase the damping of the beam by means of eddy currents. Bae et al [9] proposed a magnetically

tuned mass damper to suppress the vibration of a cantilever beam and later they used the same concept for the vibration suppression of large beam structures [10]. These studies seek to maximize the effect of eddy currents and the vibration attenuation, and so they attached coils, magnetically tuned mass dampers or conductive sheets, modifying the in this way the characteristics of the system.
The main aim of this work is to analyse the possibility of using eddy current phenomenon to attenuate vibration of beam-like structures without adding weight to the structure or modifying its stiffness. The effect of the induced eddy currents on the vibrational response of a cantilever aluminium beam is experimentally analysed in the bandwidth from 0 to 1 kHz. In addition the influence of the position of the magnetic field in the vibration attenuation of the beam is studied. Finally, a preliminary numerical model of the force generated by the eddy currents is proposed.

2. Experimental technique

The aluminium beam is tested by a forced vibration test with resonance in absence and under a partial magnetic field. Next, the tested specimen is presented and the experimental set-up is described.

2.1. Specimen

The beam is made of aluminium alloy 2024-T3. The geometrical and physical properties of the beam specimen are specified in Table 1, in which L is the free length, H is the thickness, b is the width, E^* is the complex storage modulus, ρ is the density and σ is the electrical conductivity.

Table 1. Geometrical and physical properties of the beam specimen.

L (± 0.2 mm)	H (± 0.002 mm)	b (± 0.002 mm)	E^*[1] (± 0.03 GPa)	ρ[2] (g/cm^3)	σ[2] (Ω^{-1}m^{-1})
220	0.404	9.900	66.54+i0.19	2.77	$1.75 \cdot 10^7$

1 Data obtained from experimental tests [11,12]

2 Data taken from the bibliography

2.2. Experimental set-up

In the experimental tests the transmissibility functions of the beam in a cantilever configuration in absence and under different positions of a partial magnetic field are measured. Figure 1 shows the scheme of the experimental set-up. The partial magnetic field is generated by two neodymium magnets of 50x50x8 mm placed in one side of the beam and parallel to its axis, as seen in Figure 1. The position of the magnetic field is modified by changing the distance between the magnets and the free end, x_m. The forced vibration is obtained by a base motion generated by an electrodynamic shaker. The acceleration of the base consists on a white noise in the frequency range from 0 Hz to 1 kHz. Its magnitude, $\ddot{s}$, is measured by a piezoelectric accelerometer with a charge conditioning amplifier and loopback controlled by a vibration controller. The velocity of the free-end of the beam, $\dot{u}$, is measured by a laser vibrometer located to 5 mm from the free-end. The data acquisition and signal processing are performed with an OROS analysed of four channels connected to a PC.
In the experimental tests, first the transmissibility functions in all the analysed bandwidth, from 0 to 1 kHz, are obtained. All the measurements are done in the linear range and the transmissibility function is determined relating the acceleration of the beam's free end with that applied at the base, so that

$$T^* = \frac{U^*}{S^*} \tag{1}$$

where U^* and S^* are the Fourier transforms of the derivative of the velocity measured at beam's free end, $\dot{u}$, and the acceleration applied at the base, $\ddot{s}$, respectively. Then, the resonance frequencies are identified and the modal transmissibility functions are measured in order to obtain a higher resolution. The resolution goes from 0.004 Hz in the first mode to 0.04 Hz in the last one.

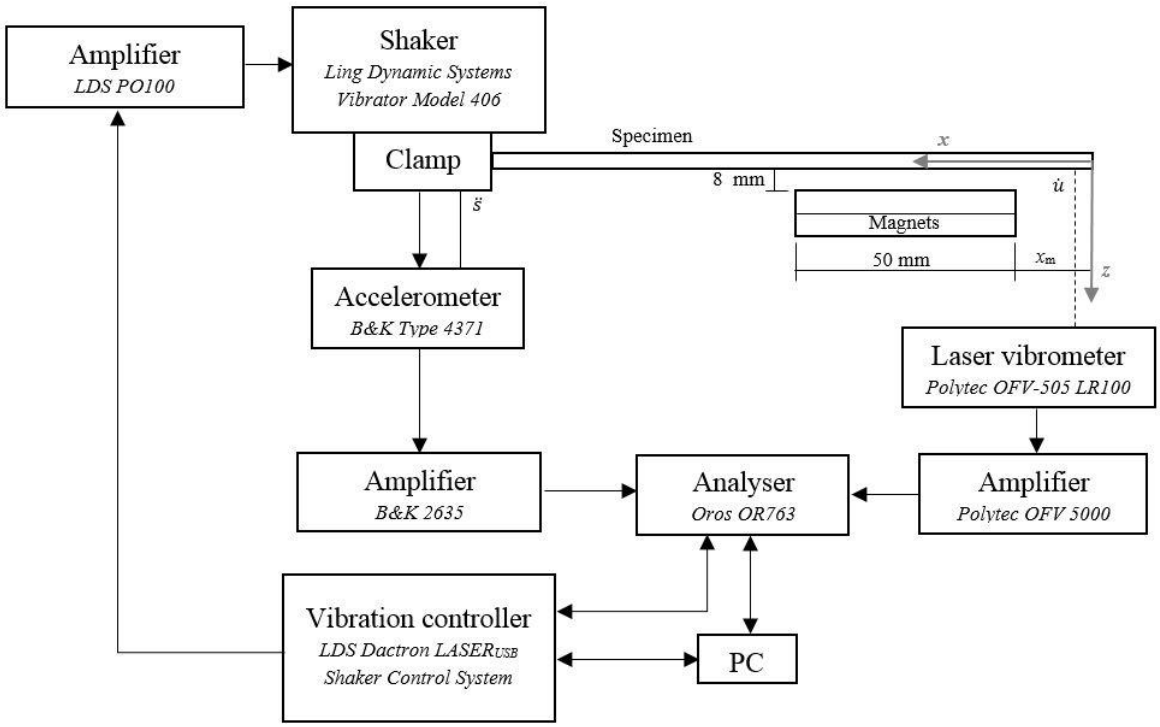

Figure 1. Experimental set-up.

3. Results

The influence of the induced eddy currents on the vibrational response of the aluminium beam is analysed. Figure 2 shows the transmissibility function of the aluminium beam in absence and under a partial magnetic field placed to x_m=8 mm in the bandwidth from 0 to 1 kHz. It is observed the natural frequencies of the system remain unmodified in all the analysed bandwidth. As the transmissibility function of all the bandwidth is shown there is no enough resolution to analyse the vibration attenuation of the beam.

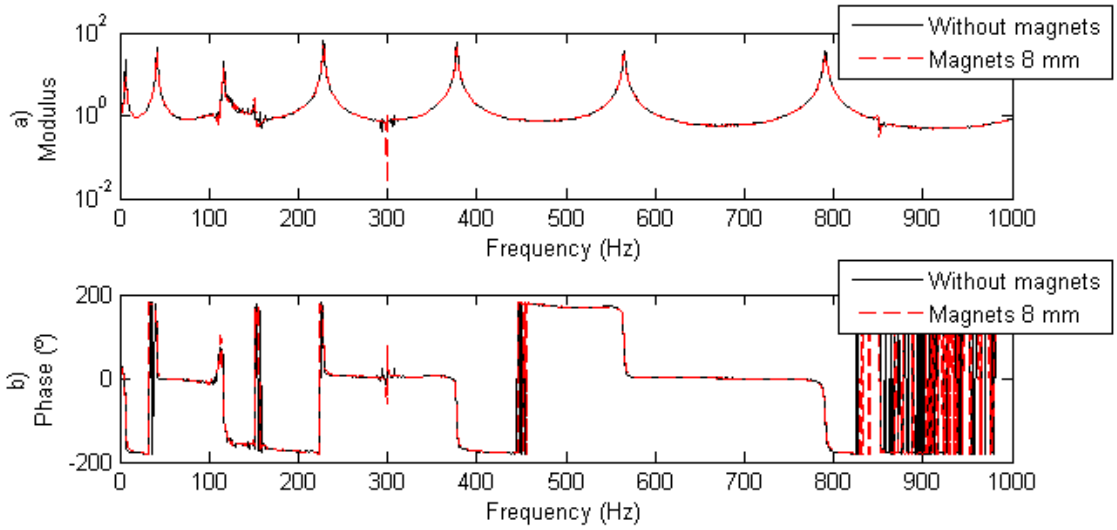

Figure 2. Transmissibility function, a) Modulus and b) Phase, of the 220 mm long aluminium beam in absence and under a partial magnetic field placed to x_m=8 mm in the bandwidth from 0 to 1 kHz.

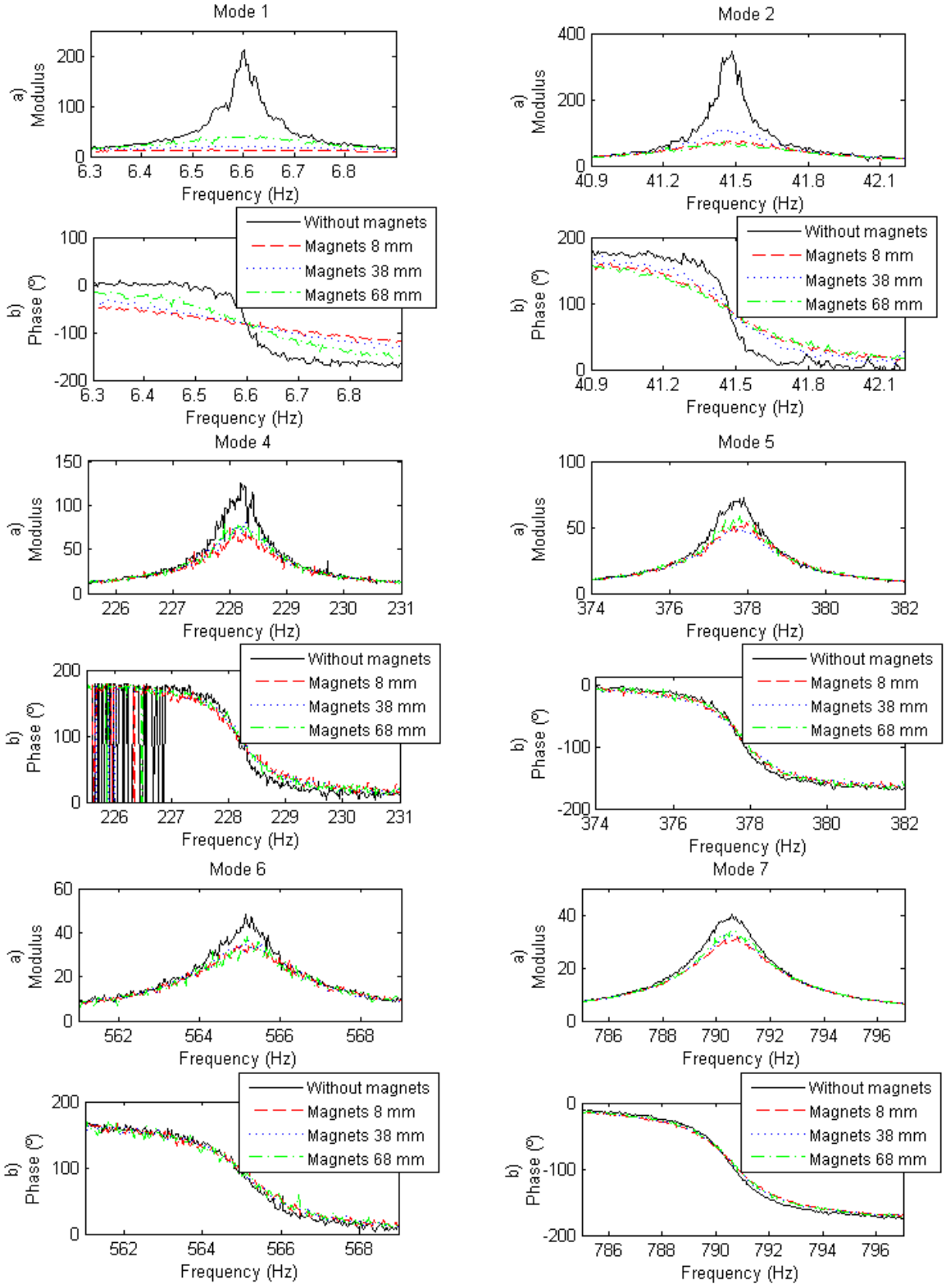

Figure 3. Modal transmissibility functions, a) Modulus and b) Phase, of the 220 mm long aluminium beam in absence and under different positions of a partial magnetic field, x_m=8 mm, x_m=38 mm and x_m=68 mm.

In Figure 3 the modal transmissibility functions of the aluminium beam in absence and under different positions of a partial magnetic field, x_m=8 mm, x_m=38 mm and x_m=68 mm, are shown. The

transmissibility modulus is related with the vibration amplitude, and so gives an idea of the damping capability of the induced eddy currents. It is observed the transmissibility modulus decreases in all the resonances when a magnetic field is applied regardless of its position. The reduction of the transmissibility modulus is more noticeable at low vibration modes, which means the eddy currents attenuates more the vibration of the beam at low frequencies. In the first two vibration modes the vibration attenuation can be even of 80%. Moreover, in the lowest vibration modes the reduction of the transmissibility modulus is affected by the position of the magnetic field.

The loss factor, η_n, is a dimensionless parameter that describes how damped an oscillator or resonator is in a vibration mode *n*. In this work the loss factor is used as a measure for quantifying the attenuation of the beam due to eddy currents and it is obtained by the half-power bandwidth (HPB) method as the standard ASTM E 746-05 recommends [11]. In the HPB method the loss factor is obtained by the division of the bandwidth where the amplitude of the peak is reduced 3 dB, Δf_n, with the resonance frequency, f_n, such that

$$\eta_n = \frac{\Delta f_n}{f_n}. \tag{2}$$

In Figure 4 the influence of the induced eddy currents on the loss factor of the beam is shown. It can be seen that applying a partial magnetic field the loss factor is increased in all the bandwidth being this tendency more pronounced in the lowest vibration modes. In addition, in this modes the loss factor of the beam is influenced by the position of the magnetic field. For example, in the first vibration mode the most effective location to attenuate vibration is x_m=8 mm, in which the loss factor is increased by 2070 %, and the less effective is x_m=68 mm, in which the loss factor is increased by 525 %. Instead, in the second vibration mode the most effective position is x_m=68 mm and the less effective one x_m=38 mm in which the loss factor is increased by 175 % and 275 %, respectively. The influence of the positon of the magnetic field has to do with the vibration shape of its mode. If the magnetic field is placed in the position where the displacement of the beam is maximum for a certain vibration mode, its influence will be maximum in that mode.

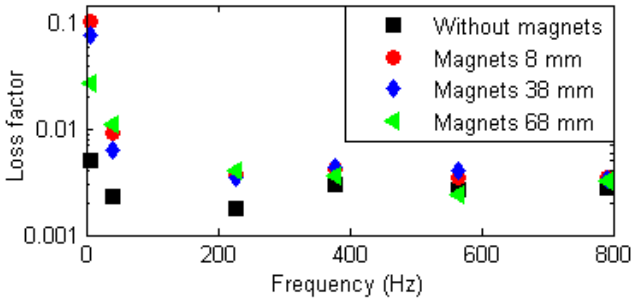

Figure 4. Loss factor of the aluminium beam in absence and under different positions of a partial magnetic field, x_m=8 mm, x_m=38 mm and x_m=68 mm.

4. Numerical modelling

Next, a numerical modelling to predict the attenuation added to a conductive and non-magnetic beam due to the induced eddy currents is proposed.

When a vibrating conductive beam is placed in a steady partial magnetic field, eddy currents are induced in the magnetic field projection area. Neglecting the surface charges, the induced eddy current density, **J**, is given by

$$\mathbf{J} = \sigma(\mathbf{v} \times \mathbf{B}) \tag{3}$$

where σ is the conductivity of the beam's material, **v** is the velocity of the beam and **B** is the magnetic field density. The beam vibrates in transverse direction, so it is assumed its velocity is only in *z* direction, such as

$$\mathbf{v} = 0\hat{\mathrm{i}} + 0\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}. \tag{4}$$

The magnetic field density can be in any direction, as follows

$$\mathbf{B} = B_x \hat{\mathrm{i}} + B_y \hat{\mathrm{j}} + B_z \hat{\mathrm{k}} . \tag{5}$$

Thus, the eddy current density can be expressed as

$$\mathbf{J} = \sigma(\mathbf{v} \times \mathbf{B}) = -\sigma v_z B_y \hat{\mathrm{i}} + \sigma v_z B_x \hat{\mathrm{j}} . \tag{6}$$

From equation (6) is deduced the eddy currents are generated in the *x*-*y* plane and the magnetic field in *z* direction does not contribute to the generation of such currents.

By the interaction of the induced currents and the magnetic flux, a damping force is generated according to Lorentz force law. It is assumed the magnetic field generated by the eddy currents is very small comparing to the external magnetic field and so it is considered the damping force comes only from the interaction between the induced eddy currents and the external magnetic flux. The damping force generated by eddy currents is obtained from

$$F_{\text{eddy}} = \int_V \mathbf{J} \times \mathbf{B} \mathrm{d}V = \int_V \left(\left(\sigma v_z B_x B_z \right) \hat{\mathrm{i}} + \left(\sigma v_z B_y B_z \right) \hat{\mathrm{j}} - \left(\sigma v_z B_y^2 + \sigma v_z B_x^2 \right) \hat{\mathrm{k}} \right) \mathrm{d}V . \tag{7}$$

It has been proved the force components in *x* and *y* directions do not affect the transverse vibration of the analysed beam. Thus, the considered eddy force is the following

$$F_{\text{eddy}} = \int_V \left(- \left(\sigma v_z B_y^2 + \sigma v_z B_x^2 \right) \hat{\mathrm{k}} \right) \mathrm{d}V . \tag{8}$$

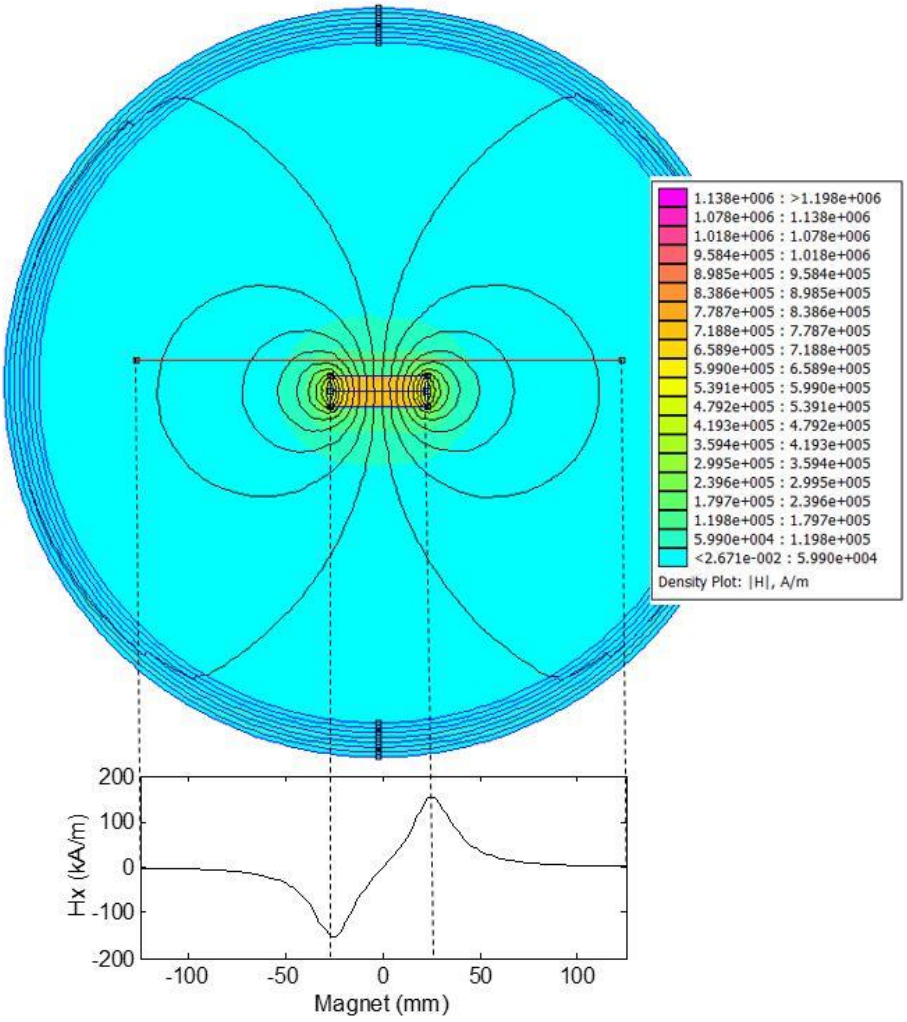

Figure 5. Simulation of the magnetic flux by FEMM 4.2 software and the magnetic field intensity in *x* direction along the length of the beam.

In this work, the partial magnetic field is obtained by placing two neodymium magnets at one side of the beam as seen in Figure 1. A simulation in FEMM 4.2 software is held to know the magnetic flux generated by such magnets configuration. The simulation parameters are the following: 2D *x-z* plane simulation, 2 NdFeB 37 MGOe magnets of 50 mm x 50 mm x 8 mm, Improvised Asymptotic Boundary Condition [13] and triangular elements with 12970 total nodes. Figure 5 shows the magnetic flux distribution and the magnetic field intensity in *x* direction, $H_x(x)$, along the length of the beam. It is observed this is maximum in the border of the magnets and almost zero in the centre. However, in order to calculate the force generated by the eddy currents, the magnetic field intensity in *x* and *y* directions, $H_x(x,y,z)$ and $H_y(x,y,z)$, must be modelled. As this can not be modelled by the simulation and may be a difficult task, in this work an inverse method is proposed to obtain the eddy force.

The inverse method is based on Finite Element Method (FEM) and experimental transmissibility functions. The transmissibility functions are measured every 5 mm along the beam in the area where a partial magnetic field is applied by the experimental technique explained in section 2. In the finite element formulation the Euler-Bernoulli beam theory is considered and it is defined by two nodes and two degree of freedom per node, the transverse displacement, *w*, and the rotational displacement, $\partial w/\partial x$.

The governing equation of motion of the beam is given by

$$[M]\{\ddot{q}\}+[K]\{q\}=\{F\} \tag{9}$$

where $[M]$, $[K]$, $\{F\}$ and $\{q\}$ are the global mass matrix, the global stiffness matrix , the global force vector and the generalized displacement vector, respectively. This can be rewritten differentiating the degrees of freedom related to the base displacement, the area where there is eddy force due to the applied partial magnetic field and the area where there is no magnetic field, indicated by $(\bullet)_s$, $(\bullet)_e$, $(\bullet)_0$ respectively, as

$$\left(-\omega^2\begin{bmatrix} M_{ee} & M_{e0} & M_{es} \\ M_{0e} & M_{00} & M_{0s} \\ M_{se} & M_{s0} & M_{ss} \end{bmatrix}+\begin{bmatrix} K^*_{ee} & K^*_{e0} & K^*_{es} \\ K^*_{0e} & K^*_{00} & K^*_{0s} \\ K^*_{se} & K^*_{s0} & K^*_{ss} \end{bmatrix}\right)\begin{Bmatrix} U^*_e \\ U^*_0 \\ S^* \end{Bmatrix}=\begin{Bmatrix} F_{eddy} \\ 0 \\ R \end{Bmatrix}. \tag{10}$$

The applied displacement at the base, $\{S^*\}$, and the displacement of the beam where a partial magnetic field is applied, $\{U^*_e\}$, are known from the experimental tests.

Frist, the displacement of the beam where it is not applied any magnetic field is calculated from

$$U^*_0=\frac{(\omega^2 M_{0e}-K^*_{0e})U^*_e+(\omega^2 M_{0s}-K^*_{0s})S^*}{-\omega^2 M_{00}+K^*_{00}}, \tag{11}$$

and then the force generated due to the induced eddy currents is obtained from

$$F_{eddy}=(-\omega^2 M_{ee}+K^*_{ee})U^*_e+(-\omega^2 M_{e0}+K^*_{e0})U^*_0+(-\omega^2 M_{es}+K^*_{es})S^*. \tag{12}$$

In Figure 6 the force generated due to the induced eddy currents in the first natural frequency can be seen. It is observed the force has a similar shape to the magnetic field H_x, as seen in Figure 5. The force is maximum in the border of the area where magnetic field is applied and smaller in the centre.

Finally Figure 7 show the experimental and numerical first modal transmissibility function in absence and under a partial magnetic field placed to x_m=8 mm. It is observed in absence of the magnetic field the finite element based on the Euler-Bernoulli beam theory is able to reproduce the behaviour of the beam. The eddy force has been obtained by the inverse method for 5 different frequencies: 8 Hz, 9 Hz, 9.85 Hz, 11 Hz and 12 Hz. It is observed applying these forces the transmissibility modulus agrees with the experimental one under a partial magnetic field.

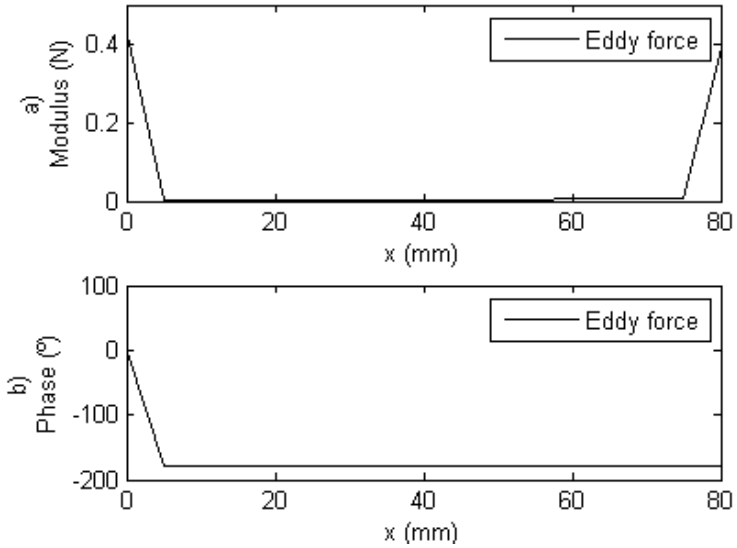

Figure 6. Eddy force along the length of the beam where magnetic field is applied obtained by the inverse method in the first natural frequency.

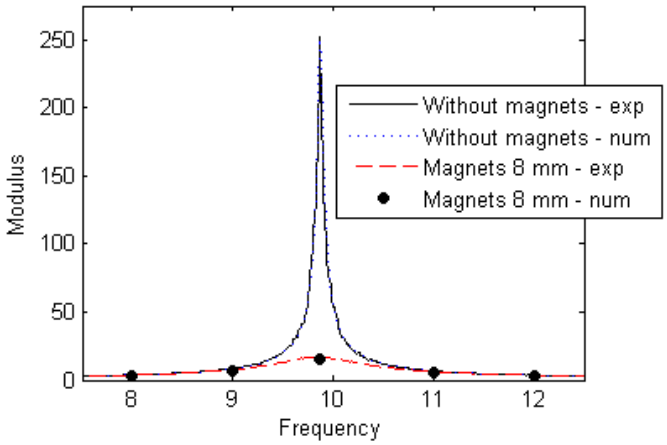

Figure 7. Experimental and numerical transmissibility function of the first natural frequency of 180 mm long beam in absence and under a partial magnetic field placed to x_m= 8 mm.

5. Conclusions

In this work the possibility of using the eddy currents phenomenon to attenuate the vibration of conductive and non-magnetic beam-like structures without adding weight to the structure or modifying its stiffness is studied. The influence of the induced eddy currents on the vibrational response of an aluminium cantilever beam is experimentally analysed in the bandwidth from 0 to 1 kHz and a preliminary numerical model for the eddy force is proposed and validated in the first vibration mode. The natural frequencies of the beam are not modified by the induced currents and the vibration is attenuated in all the analysed bandwidth, being this tendency more remarkable at low frequencies. This result emphasizes the possibility of attenuating vibration of conductive and non-magnetic beams without changing the system itself and its properties. In addition, the position of the magnetic field affects the vibration attenuation of the beam, and the vibration reduction is maximum when the partial magnetic field coincides with a maximum vibration of a mode.

6. References

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